CLASSROOM ACTIVITIES: APPLICATIONS OF SYNTHETIC DIVISION

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Synthetic Division. Synthetic division as it is usually taught involves division of polynomials by first degree monic polynomials. These are polynomials of the type $x + c$. But the synthetic division algorithm can be extended to division by polynomials of any degree.

Monic Polynomial Divisors. The algorithm is best shown by illustration.
Example: Divide $2x^5 - 3x^4 + x^2 - 7x + 2$ by $x^2 - x + 2$.

The first step is to set up the tableau:

$$\begin{array}{rr|rrrr|rr} 1 & -2 & 2 & -3 & 0 & 1 & -7 & 2 \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \end{array}$$

As in division by first degree monic polynomials, the coefficient of the leading term of the divisor is ignored and the remaining coeffients are represented on the top line of the tableau by their negatives. Following the separator bar are the coefficients of the dividend. The last two coefficients of the dividend are separated from the rest to mark the location of the remainder.

The division process begins when the leading coefficient of the dividend is copied to the last line:

$$\begin{array}{rr|rrrr|rr} 1 & -2 & 2 & -3 & 0 & 1 & -7 & 2 \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & \underline{\mathbf{2}} & & & & & \end{array}$$

Next, this number is multiplied by the numbers in the divisor column and the result displayed beginning in the next column:

$$\begin{array}{rr|rrrr|rr} 1 & -2 & 2 & -3 & 0 & 1 & -7 & 2 \\ \hline & & & \underline{\mathbf{2}} & \underline{\mathbf{-4}} & & & \\ \hline & & & & & & & \\ \hline & & 2 & & & & & \end{array}$$

Next, the sum of the numbers in the second dividend column is entered below the line:

$$\begin{array}{rr|rrrr|rr} 1 & -2 & 2 & -3 & 0 & 1 & -7 & 2 \\ \hline & & & 2 & -4 & & & \\ \hline & & & & & & & \\ \hline & & 2 & \underline{\mathbf{-1}} & & & & \end{array}$$

The sum of the second dividend column is multiplied by the numbers in the divisor columns and the result displayed beginning in the next column:

1	−2	2	−3	0	1	−7	2
			2	−4			
				−1	**2**		
		2	−1				

The third column of the dividend is totaled:

1	−2	2	−3	0	1	−7	2
			2	−4			
				−1	2		
		2	−1	**−5**			

The sum of the third dividend column is multiplied by the numbers in the divisor columns and the result displayed beginning in the next column, reusing the first line:

1	−2	2	−3	0	1	−7	2
			2	−4	**−5**	**10**	
				−1	2		
		2	−1	−5			

The last of the non-remainder dividend columns is totaled:

1	−2	2	−3	0	1	−7	2
			2	−4	−5	10	
				−1	2		
		2	−1	−5	**−2**		

and multiplied by the divisor columns:

1	−2	2	−3	0	1	−7	2
			2	−4	−5	10	
				−1	2	**−2**	**4**
		2	−1	−5	−2		

The final step is to total the two remainder columns:

1	−2	2	−3	0	1	−7	2
			2	−4	−5	10	
				−1	2	−2	4
		2	−1	−5	−2	**1**	**6**

The totals of the first four dividend columns are the coefficients of the quotient and the totals of the two remainder columns are the coefficients of the remainder.

Thus $Q(x) = 2x^3 - x^2 - 5x - 2$ and $R(x) = x + 6$.

Application to evaluation of complex functions.

Example: Given $f(z) = 3z^4 - z^3 + 5z^2 - 3z + 2$, find $f(2 - i)$.

Both $2 - i$ and its conjugate $2 + i$ are zeros of the polynomial $[z - (2 - i)][z - (2 + i)] = z^2 - 4z + 5$. So if $f(z)$ is divided by $z^2 - 4z + 5$ to find the first degree remainder $R(z)$ then $f(2 \pm i) = R(2 \pm i)$.

4	−5	3	−1	5	−3	2
			12	−15	136	−170
				44	−55	
		3	11	34	78	−168

Since $R(z) = 78z - 168$ it follows that $f(2-i) = R(2-i) = 78(2-i) - 168 = -12 - 78i$.

Powers of complex numbers.

We customarily teach students to find powers of complex number using DeMoivre's theorem. For positive integral powers, however, we can also use synthetic division.

Example: Find $(1-2i)^6$

We divide z^6 by $[z-(1-2i)][z-(1+2i)] = z^2 - 2z + 5$ using synthetic division as follows:

$$\begin{array}{rr|rrrrr|rr}
2 & -5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
 & & & 2 & -5 & -2 & 5 & -38 & 95 \\
\hline
 & & & & 4 & -10 & -24 & 60 & \\
\hline
 & & 1 & 2 & -1 & -12 & -19 & 22 & 95
\end{array}$$

Thus, $R(z) = 22z + 95$ so $(1-2i)^6 = R(1-2i) = 22(1-2i) + 95 = 117 - 44i$.

This computation is much easier than the DeMoivre's theorem computation.

Application to surds.

Example: $f(x) = 2x^4 - 5x^3 + 2x - 4$. Find $f(1-2\sqrt{3})$.

Find the remainder $R(x)$ after division by $[x-(1-2\sqrt{3})][x-(1+2\sqrt{3})] = x^2 - 2x - 11$.

$$\begin{array}{rr|rrr|rr}
2 & 11 & 2 & -5 & 0 & 2 & -4 \\
\hline
 & & & 4 & 22 & 40 & 220 \\
\hline
 & & & & -2 & -11 & \\
\hline
 & & 2 & -1 & 20 & 31 & 216
\end{array}$$

Thus $f(1-2\sqrt{3}) = 31(1-2\sqrt{3}) + 216 = 247 - 62\sqrt{3}$.

For good students needing a little more than the normal fare, teachers might consider showing them general synthetic division and its applications.

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