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Robert Recorde
"With the helpe of it, you maie attayne to all thyng." A practical application of mathematics.

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Robert Recorde (c.1512-1558) has long been seen by historians and mathematicians as being significant for his works on the practical teaching of mathematics to a non-specialized audience. What we are trying to show in this essay is how Recorde, precisely because of his role as a teacher, came up with a practical, useful, and educationally workable way of presenting mathematical concepts to a general audience. Specifically we have looked at two of his works, *The Grounde of Artes* and *The Whetstone of Witte*, to show and analyze his teaching pedagogy. We also discuss the different audiences Recorde apparently had in mind for these works. While *The Grounde of Artes* seems to be aimed at merchants, *The Whetstone of Witte* has numerous examples related to the utility of mathematics to the military.

Keywords: Robert Recorde, Sixteenth century Mathematics, England

The sixteenth century mathematician and physician Robert Recorde remains a curious and enigmatic figure in the history of those disciplines. Although Recorde was not the first to publish a mathematical work in English (Roberts, 2016, p. 35), his claim to fame rests on the simple fact that his work is regarded as the best early English mathematical work to be published in English, rather than the academic language of Latin. Accompanying factors that give him a nod were that he is recognized as the first mathematician to use the symbols + and − in a text published in English, and the first to introduce the = sign in any language[1]. Beyond these facts what lasting or monumental contributions did Recorde make to the field of mathematics? Clearly Recorde comes across more in the image of a teacher rather than a scholar of merit. For these reasons, until more recent times, Recorde has been considered a minor figure in the history of mathematics. He made no lasting contribution to the advancement of mathematics or physiology. Where his importance or relevance rests is in his capacity as a teacher and writer. Particularly in the addressing of his works, not to a learned academic audience, where Latin would be the language of discourse, but rather focusing his writings to a more pragmatic commercial or popular audience. In this regard one certainly gets the impression that his concerns centered on the functionality of mathematics in achieving tangible measurable ends. The abstract and theoretical aspects of mathematics did not seem to concern him to any great degree — at least not in his published works. His designs were clearly aiming at a more utilitarian usage of mathematics.

Two of Recorde's works of particular interest in his reaching out to a non-academic audience are his first published work, completed in 1543, *The Grounde of Artes*, and his last published work, *The Whetstone of Witte*, published in 1557. In both of these works we see Recorde applying the Socratic Method in working through his arguments in the form of dialogue between Master and Scholar. John Aubrey, in his *Brief Lives*, stated of Recorde that:

> He was the first that wrote a good arithmetical treatise in English, which hath been printed a great many time...his Arithmatick, containing the ground of arts in which is taught the general parts rules and operations of the same in whole numbers and fractions after a more easie and exact method then ever heretofore, first written by Robert Recorde... (Aubery, 1898, p. 198-199)

What is of particular interest are the choices he uses in the explanations he provides to clarify a specific point.

As Joy Easton has stated in regard to *The Grounde of Artes*, it was "the most popular Arithmetic of the Sixteenth century" (Easton, 1967). From the start of Recorde's career as a published author of mathematics it is clear that his designs were not to strike new ground in the field of mathematics, but rather to make an appeal to the uneducated in order to show the usefulness that mathematics possessed. Clearly Recorde sacrificed a European wide reputation as a scholar of note by publishing his works in English rather than in Latin. Here we see Recorde coming across as an educator particularly interested in the teaching of applied mathematics, with a

[1] Johannes Widmann of Eger had used the plus and minus signs in his text of 1489 in German (Widmann, 1489). Cf (Reich, 2012, p. 102).

clear design to meet the needs and interests of the merchant community. We can see through all of Recorde's works an emphasis being placed on the utility of mathematics (Reich, 2012, p. 116).

What comes across with stunningly simple clarity, and shows Recorde as the superlative educator, is both the style and format of his presentation to the reader. Also, the simple, straight forward examples that he presents to elaborate and clarify point by point through Socratic dialogue in discussion between Master and Scholar, is lucid and practical (Denniss & Smith, 2012, p. 26). What comes across is an example of the quintessential teacher. Oftentimes scholarship and teaching abilities are not seen as being of equal merit. Scholarship always trumps mere teaching, and perhaps that is the natural order of things. However if a scholar lacks the ability to convey his scholarship then of what use is it? Recorde's great gift, we would argue, was to be able to elucidate complex nuances, to cut away complications, and make the material discernable to the average reader.

At the beginning of *The Grounde of Artes Recorde* stresses the all-encompassing importance of mathematics when he says: "wherfore as without nomberynge a man can do almost nothynge, so with the helpe of it, you maye attayne to all thyng" (Recorde, 1543, p. 20). He then goes on to stress the importance of numbers and arithmetic and their applications to a variety of fields and pursuits–such as astronomy, geometry, music, physics, the law, grammar, philosophy, government, military arts, and business. As the Master sums up in his discussion: "Yf I shulde (I saye) particularly repete all such commodities of this noble science of Arithmetike, it were ynough to make a very greate booke." The Scholar then concurs and responds: "I dowte not, for this that you haue sayd, were enowgth to perswade any manne, to thynke this arte to be right excellente and good, and so necessarie for man, that (as I thynke nowe) so moch as a man lacketh of it, so moche he lacketh of his sense and wytte" (Recorde, 1543, p. 24-25). That thesis is then supported throughout the rest of the book through numerous examples. What Recorde emphasizes is the practical importance of mathematics. As he stated: "For it is often practise that maketh a man quycke and rype in all thynges" (Recorde, 1543, p. 62). And "Howbeit I wyll yet exhorte you now to remember both this, that I haue said, and all that I shall saye, and to exercyse your selfe in the practise of it: for rules without practise, is but a lyght knowledge, and practise it is, that maketh men pfecte and prompte in all thynges" (Recorde, 1543, p. 44).

Whether at present, or in the sixteenth century, the dedicated teacher has a love for their subject matter and a general love of learning. As Recorde puts it: "What treasure is there to be compared to the ryches and treasure of lernynge?" (Recorde, 1543, p. 99). This simple statement succinctly makes the point. Recorde goes on in stressing the importance of using language that the students can understand. Clearly sound advice for any generation. Recorde has the Scholar agree when he says: "Mayster I thanke you hartely, I perceaue you seke to instructe me moste playnely and brefly, and not to hyde your knowlege with subtyll wordes, as many do" (Recorde, 1543, p. 48).

Perhaps the greatest challenge for any teacher is to protect their students from their own personal prejudices, to encourage a student to question and test out their own hypotheses, rather than mutely and blindly accepting the word of the teacher. Here the point is made by Recorde when the Scholar tells the Master: "What so euer you say, I take it for trewe." [A position, still to this day, that many teachers would tend to agree with!] Recorde, however, rather shockingly, has the Master reply: "That is to moch, and mete for no man, to be beleued in al thynges, without shewynge of reason. Though I myght of my Scoler some credence requyre, yet except I shewe reason, I do not it desyre" (Recorde, 1543, p. 26).

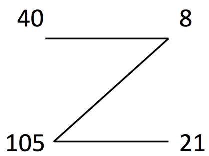
Recorde's teaching style is concerned with understanding basic points before moving on to more complex materials (Johnson & Larkey, 1935, p. 70). As Recorde has the Master state the point to the Scholar: "What I declare by one example, do you expresse by an other, and so shall I perceaue whether you understand it or no. And so passe ouer nothynge tyll you perceaue it well, and be experte therin." (Recorde, 1543, p. 32) Here we see a vivid example of mastery learning. Notice how Recorde has the Master point out common mistakes so that the reader can avoid them: "M. How write you these foure other .ii. i. ix. viii? S. Thus (I trowe) 2. 1. 6. 8. M. Nay, there you mysse: Loke on myne example agayne. S. Syr trouth it is, I was to blame, I toke 6 for 9, but I wyll be warer here after" (Recorde, 1543, p. 33-34).

In *The Grounde of Artes*, Recorde seems to be targeting the merchant class. He makes this clear in the preface of *The Whetstone of Witte*, when he refers back to his previous book of arithmetic. Lamenting the lack of willingness of the general public to study mathematics, Recorde writes, "Many praise it, bue fewe dooe greately practise it: onlesse it bee for the vulgare practice, concernyng Merchaundes trade. Wherein the desire and hope of gain, maketh many willyng to sustaine some trauell. For aide whom, I did sette forth the firste parte of Arithmetike" (Recorde, 1557, p. 13). It becomes even clearer who Recorde's intended audience is when we observe the examples he uses to clarify his teaching.

One of the topics treated by Recorde is that of proportions. In *The Grounde of Artes*, the Master poses the following question to the Scholar: "But and the questyon be asked thus: In 8 wekes I spend 40 S. howe longe wyll 105 S. serue me?" The Master then proceeds to explain to the Scholar how to set up the proportion: Because 40 and 105 are of the "same denomination" (shillings), they are placed in one column. The other number involved, 8, is of a second denomination(weeks), so is placed in the top position of a second column. The Master then instructs the Scholar with the arithmetic involved to find the second number for this column:

> Then multiply 105 by 8, and it wyll be 840, whiche yf you diuide by 40, it wyll yelde 21, whiche is the fourth nomber, and showeth how many wekes 105 S. wyll serue, yf you spende 40 S. in 8 wekes. The figure of this questyon is this, as yf you shuld say: yf 40 S. serue for 8 wekes, 105 serue for 21 wekes (Recorde, 1543, p. 234).

Here is the diagram that Recorde uses to illustrate the process:



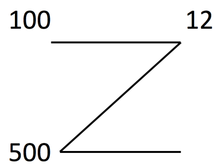
In modern notation, we would set up the proportion as

$$\frac{40}{105} = \frac{8}{x},$$

and then cross-multiply to solve for the unknown.

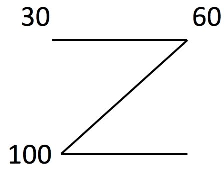
Recorde then teaches the reader how to handle problems that require multiple proportions. As the Master asks, "Yf the caryage of 100 pound weyghte 30 myles, do coste 12 d. how much wyll the caryage of 500 weyghte coste, beynge caryed 100 myles?" The Master then explains the process: First set up a proportion to determine the cost of carrying 500 pounds for 30 miles. He instructs the Scholar as follows:

> Set your figure thus,



> and multyply 500 by 12, and therof amounteth 6000, whiche yf you diuide by 100, the quotient wyll be 60, that is the pryce of 500 for 30 myles. Then begyn the second worke sayenge: yf 30 myles cost 60 d. how moche wyll 100 myles cost?

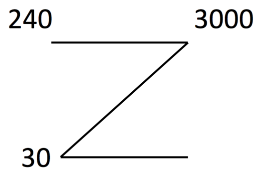
Recorde gives the following diagram:



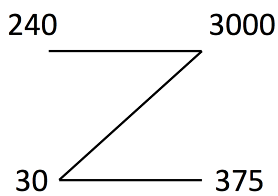
He then solves this second proportion to obtain the final cost of 200 d (Recorde, 1543, p. 239).

The following example instructs merchants in how to divide their profits: "Foure marchauntes of one company made a bancke of monye dyuersly, for the fyrste layde in 30 li. the seconde 50 li. the thyrde 60 li. and the fourth 100 li. whiche stocke they occupyed so longe, tyll it was encreaseth to 3000 li. Now I demaund of you, what shulde eche man receaue at the partynge of this monye?" The Master then instructs the Scholar to add the four sums of money that the merchants invested to obtain 240 li. Then he explains how to use the rule of proportions, which Recorde calls the "golden rule."

> The parcels of those foure marchauntes make in one summe 240 li. Set that in the fyrst place, the gaynes in the seconde, and the fyrste mans portion of stocke in the thyrde place, thus,

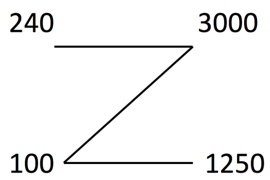


> Now multyply the seconde by the thyrde, and it wyll bee 90000, which you shal diuide by 240, and there wyll appere 375 li. thus,

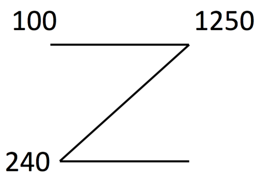


> and that is the gaynes for the fyrst man.

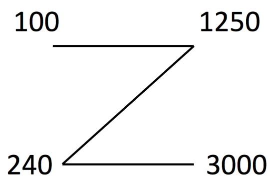
The Master then helps the Scholar to discover, via three similar proportions, that the second, third, and fourth merchants should receive 625 li, 750 li, and 1250 li, respectively. The Scholar, seeming unsure of the correctness of his computations, then asks the Master, "This I perceaue, but is there any way to examyn whether I haue wel done or no?" The Master replies, "That must you do by one commyne profe, which serueth to the golden rule, and all other ensewynge of the same." The Master then tells the Scholar how to check his work by changing the placement of the numbers in a proportion. As an example, he takes the proportion that was used to determine the gains that the fourth merchant made,



and changes it to



This new proportion is then solved to obtain



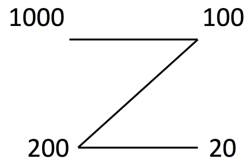
which verifies the original solution of 1250 (Recorde, 1543, p. 245-247).

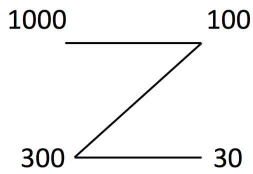
Recorde then instructs merchants in how to divide losses:

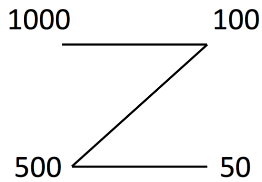
> Well, now an other example wyll I put to you not of gaines, but of losse: for one reason serueth for bothe. Yf thre marchauntes in one shyppe, and of one felowshyp, had bought marchaundyse, so that the fyrste had layd out 200 li. the seconde 300 li. and the thyrde 500 li. And it chaunced by tempest that they dyd cast ouer borde into the see marchaundyse of the valewe of 100 li. how moch shuld eche man bere in this losse?

Recorde uses the Scholar's reply to demonstrate the usefulness of examples in learning mathematics:

> If I shall do in this as you dyd in the other questyon, then must I ioyne theyr thre portions together, 200, 300, and 500 whiche maketh 1000: then say I, yf 1000 lese 100, then shal 200 lese 20, and 300 shall lese 30, and 500 shall lese 50, as by these .iii. figures it doth appeare playne.







Note that Recorde has the Scholar find the solution independently, as opposed to the previous problem, where the Master walks him through the process step-by-step. (Recorde, 1543, p. 248-249)

Whereas *The Grounde of Artes* presents basic notation and simple arithmetic, in *The Whetstone of Witte*, Recorde's last published work, he delves into the more complex world of algebra. Aside from a more sophisticated level of mathematical knowledge that Recorde is attempting to impart, the teaching method remains consistent with all his earlier works. Foremost in that teaching method is the fact that he continues to impart his knowledge using English. As he stated:"In this booke what I haue written, for the aide of all menne, and namely soche of my countrie menne, that understand nothyng but Englishe...in this I am the first venturer" (Recorde, 1557, p. 13-14). He also presses the point that it is the obligation of learned people to share their knowledge with the less educated. On this point Recorde makes a departure from the Renaissance notion that true learning was for the adept and not to be shared with the masses. As he expounds:

> Doeth it excite the beste learned, to remember their duetie to their countrie: and to be a shamed, that thei hauyng so greate habilitie, shall be founde moare slacke to aide their countrie, then he that hath smaller knowledge, and lesse occasion otherwaies. Accordyng as men haue re-

> ceiued, so are thei bounde to yeld. These excellente giftes are not lente unto men, to be hidden. And there are a great multitude that thurst, and long moche for soche aide (Recorde, 1557, p. 3-4).

This idea is a century ahead of Francis Bacon's similar notion on the utility of knowledge.

He continues on the idea of the importance on knowledge when he has the Scholar ask of the Master:

> We shall not nede to stande on this talke, but trauell with knowledge to vanquishe ignoraunce: And beleue that the pricke of knowledge, is more of force then the stynge of ignoraunce: yea, the pointe in Geometrie, and the unitie in Arithmetike, though bothe be undiuisible, doe make greater woorkes, and increase greater multitudes, then the brutishe bande of ignoraunce is hable to withstande. (Recorde, 1557, p. 17-18)

Or, as Recorde said more succinctly: "Learnyng bee moste to be loued, for knowledges sake." (Recorde, 1557, p. 267)

More specifically, concerning mathematics and numbers, Recorde emphasizes the importance of mathematics in shaping intellect. As he puts it:

> But yet one commoditie moare, whiche all menne that studie that arte, doe fele, I can not omitte. That is the filyng, sharpenyng, and quickenyng of the witte, that by practice of Arithmetike doeth insue. It teacheth menne and accustometh them, so certainly to remember thynges paste: So circumspectly to consider thynges presente: And so prouidently to forsee thynges that followe: that it maie truelie bee called the File of witte. Yea it maie aptly bee named the Scholehouse of reason. (Recorde, 1557, p. 12)

Clearly mathematics is seen as the most important key for the honing of one's mind. All other knowledge is simply borrowed. As Recorde puts it: "Nomber onely maketh all artes perfecte, and worthie estimation: seyng that without it, all artes are but base, and without commendation." (Recorde, 1557, p. 12)

Recorde is ever the consummate teaching advocate always wanting the student to be able to work their way logically through every problem with little "hand-holding." As he has the Master put it: "But for euery mater to require aied, and neuer to trauell my owne witte, it might seme mere dastardlinesse. And so were it plaine babishenesse, to couet euery morsell, to be chawed before hande, and put into my mouthe" (Recorde, 1557, p. 185-186). He stresses the idea of encouraging the student and using examples for clarification on difficult points. Or as he has the Master put it: "For examples are the lighte of teachyng." (Recorde, 1557, p. 71)

In *The Whetsone of Witte* Recode provides over fifty examples to clarify the points that he is making. Of these examples over half are word problems of which a dozen problems concerning military usage. The remainder are concerned with other practical matters regarding business, building, and proportioning.

Recorde concurs with Plato's assessment of the importance of mathematics to the military arts:

> Plato thinketh noe manne hable to bee a good capitaine, excepte he bee skifulle in this arte...Howbeit for the better trialle thereof, I haue in this Booke framed some of the questions in soche sorte, as thei maie approue the use of this arte, not onely good for capitaines, but also moste necessarie for theim. So that without it, thei can not Marshall their battaile, nother vewe their enemies campe or forte. And if I shall saie as I thinke, without it a capitaine is noe capitaine (Recorde, 1557, p. 13).

Recorde makes it clear that he believes the mathematics contained in *The Whetsone of Witte* far exceeds that of his previous book of arithmetic. Speaking of the readers of *The Grounde of Artes*, he says:

> But if thei knewe how farre this seconde parte, dooeth excell the firste parte, thei would not accoumpte any tyme loste, that were imploied in it. Yea thei would not thinke any tyme well bestowed, till thei had gotten soche habilitie by it, that it might be their aide in al other studies. (Recorde, 1557, p. 13)

One of the skills necessary for military usage was that of being able to extract square roots. This was quite an involved process 500 years ago, and Recorde devoted over fifty pages to it. Let's discuss the procedure by finding the square root of 104,976. We set up the problem like this:

$$1\underset{\cdot}{0}4\underset{\cdot}{9}7\underset{\cdot}{6}\ ($$

Recorde uses dots, which he calls "prickes," to keep track of the steps of the process, and also to determine how many digits the answer should have. We start at the right of the number, and place a dot under every other number. So in this example, the 6, the 9, and the 0 have dots under them. From this, we know that our answer should be a three-digit number. Then we start with the number that occurs up to the first prick, 10. The largest perfect square that is less than or equal to 10 is 9. The square root of 9 is 3, which we place to the right of the parenthesis.

Then we subtract 9 from 10 to get 1, and replace the 10 with 1 in our problem:

14976(3

Then there are four steps that we will repeat until the final answer is obtained: doubling, dividing, squaring, and subtracting.

1. Double the square root, 3, to get 6.

2. Divide the first two digits of the number, 14, by 6, to obtain 2, with a remainder of 2. The quotient, 2, becomes the next digit of the square root, and the remainder, 2, replaces the 14 in the problem:

2976(32

3. Now square the 2 to get 4.

4. Subtract 4 from 29 to get 25, and replace 29 with 25.

2576(32

Now we repeat this process:

1. Double the square root, 32, to get 64.

2. Divide 257 by 64 to get 4, with a remainder of 1. The quotient, 4, becomes the next digit of the square root, and the remainder, 1, replaces the 257 in the problem:

16(324

3. Square 4 to get 16.

4. Subtract 16 from 16, to get 0. So the square root of 104,976 is 324.

To show how the process of extracting square roots might be used, Recorde poses the following:

> question of an armie: A Prince hath an armie verie greate. With whiche he passeth in a Vallie, so that in marchynge the fronte can be but. 18. menne. And by that meanes the flancke containeth. 449352. After that the armie is passed that valie, the kyng mindyng to occupie all the beste grounde, willeth the battaile to be set square. How would you doe it?

The Scholar proceeds as follows: "First I multiple flancke, by the front. And so I finde the whole nomber to be. 8088336." This is the total number of soldiers. To determine how to form the soldiers into a square battalion, Recorde has the Scholar use the method presented by the Master to compute the square root of 8088336 as 2844 (Recorde, 1557, p. 109-110).

Recorde also gives a method for extracting cube roots. It is similar to the procedure for square roots, but significantly more involved. He first gives a military example that uses the operation of finding the cube of a number in the following "question of a Gonne: A Gonne of sixe inches diameter in the mouthe, doeth shotte a bollet of twentiepound weighte: what weighte shall that bollette haue, that serueth for a gonne of. 14. inches in the mouthe?" The Master, instructing the Scholar in the solution, quotes the 18^{th} proposition of the 12^{th} book of Euclid: "All Globes here together triple that proportion, that their diameters doe." This means that we have to cube the ratio of the two diameters to get the ratio of the weights, because weight is directly proportional to volume. The Master takes the ratio of the diameters, $\frac{14}{6}$, and reduces it to $\frac{7}{3}$. He continues: "Wherefore I sette the .3. fractions thus, $\frac{7}{3}\frac{7}{3}\frac{7}{3}$and thei make $\frac{343}{27}$. that is. $12.\frac{19}{27}$. And so is the proportion of the Globes, as well in weighte, as in bignesse." To find the weight of the larger "bollette", he multiplies 20 by 343, and then divides the result, 6860, by 27, to obtain $254\frac{2}{27}$ pounds (Recorde, 1557, p. 136-137).

Next, in order to apply the practice of taking cube roots, Recorde gives a similar problem, but requiring the inverse operation, or "in backer order," as he puts it. "A bollette of yron of. 7. inches diameter, doeth waie 27. pounde weighte: what shall be the diameter to that bollette that shall waie. 125. pounde weighte?" The Scholar, unsure of his abilities, is hesitant to attempt a solution: "I praie you aunswer to it your self, that I maie see the apte forme of appliyng soche questions to this rule." The Master obliges: "As the Cubes are in triple proportion to the sides, so are the proportions of the sides, to bee founde by triple diuision: that is to saie, by seking the Cubike rootes, of the 2. termes of the proportion." He sets up the ratio of the two weights as $\frac{125}{27}$. Then, taking the cube roots of both numerator and denominator, the ratio of the diameters is found to be $\frac{5}{3}$. Then he multiplies the smaller diameter, 7 inches, by 5, and divides the result by 3 to get $11\frac{2}{3}$ inches. "Wherefore I saie, that if. 7. inches bee the diameter to a bollette of. 27. pounde weighte, then. 11. inches and $\frac{2}{3}$ shall be the diameter to the bollete of. 125. pounde weighte." The Scholar seems a bit unsure of the procedure: "The proofe of this had nede bee certain, seeyng the woorke is obscure, to the common iudgemente." Recorde has the Master provide the following method to check the veracity of the solution. Take the diameters, 7 and $11\frac{2}{3}$, and cube each of them. The ratio of these cubes, once reduced, should be $\frac{125}{27}$ (Recorde, 1557, p. 137-139).

In the next chapter, Recorde introduces his readers to the wonderful world of algebra, or "The Arte of Cossike nombers," (Recorde, 1557, p. 150) as he calls it. Cossike numbers are variables, and he gives a different symbol for every power of a variable. For example, if we are using the letter x for our variable, then $x_0 = 1$ is symbolized by [symbol], x^1 by [symbol], x^2 by [symbol], x^3 by [symbol], and x^4 by [symbol]. After introducing these symbols, Recorde demonstrates how to add, subtract, multiply, divide, and extract roots of algebraic expressions. Once these operations are mastered, with the aid of numerous examples, he moves on to algebraic equations, or "the rule of equation, commonly called Algebers Rule" (Recorde, 1557, p. 240). Of this rule, Recorde says that "by equation of nombers, it doeth dissolue doubtefull questions: And unfolde intricate ridles" (Recorde, 1557, p. 240). It is in this section that Recorde introduces the symbol that we still use for "is equal to." He explains: "And to auoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: = bicause noe. 2. thynges, can be moare equalle" (Recorde, 1557, p. 242). Note that when Recorde first introduced the symbol, it was much longer than what we use today, for example:

$$\overline{\underline{\hspace{4cm}}}.$$

Recorde then gives several examples of equations, followed by some guidelines to follow when solving them: "If you abate euen portions, from thynges that bee equalle, the partes that remain shall be equall also" (Recorde, 1557, p. 243). And, "If you adde equalle portions, to thynges that bee equalle, what so amounteth of them shall be equalle" (Recorde, 1557, p. 243). In other words, you can add or subtract the same quantity from both sides of an equation, and the equality remains. Then he presents two types of equations. The first type is equations with two terms. He gives examples that, in modern mathematical notation, would be written as $4x^2 = 12x$, $14x^3 = 70x^2$, and $15x^5 = 90x^4$. In each of these examples, the exponents of the two terms differ by one. He explains how to solve these by dividing both sides of the equation by the greatest common factor of the two terms. For example, he solves $4x^2 = 12$ by dividing both sides by $4x$ to obtain $x = 3$. Of course, the solution $x = 0$ is lost in this process.

Then he mentions "The seconde forme of the firste equation," (Recorde, 1557, p. 247) in which the exponents differ by more than one. He gives examples like $6x^3 = 24x$, $7x^5 = 576x$, and $7x^5 = 56x^2$. The process he gives starts out the same as before, but it is followed by the extraction of a root. For example, $6x^3 = 24x$ is tackled by dividing both sides by $6x$ to obtain $x^2 = 4$, and taking the square root of 4 to get $x = 2$. Here the solutions $x = 0$ and $x = -2$ are not mentioned by Recorde.

Now "the seconde kinde of equation" (Recorde, 1557, p. 248) is introduced. This type has three terms. There are two forms. In the first form, the exponents of the three terms are sequential. Recorde gives examples like $4x^2 = 6x + 4$, $6x^5 = 12x^4 + 18x^3$ and $2x^2 = 120 - 8x$. In the second form, the exponents are not sequential. Some examples given are $5x^4 = 60x^2 + 320$, $8x^6 = 40x^3 + 30208$, and $9x^7 = 90x^4 - 144x$. Recorde uses a method similar to completing the square to find solutions to these equations, but solutions that are negative or complex are not given.

After the Master presents these different types of equations, the Scholar makes a request: "I dooe couette some apte questions, appertainyng to these equations" (Recorde, 1557, p. 250). Recorde obliges by presenting several word problems that utilize the algebra he has previously taught. Recorde asks the following:

> question of an armie: There is a capitain, whiche hath a greate armie, and would gladly Marshall them, into a square battaile, as large as mighte bee. Wherefore in his firste proofe of square forme, he had remainyng. 284. to many. Any prouyng again by puttyng. 1. moare in the fronte, he founde wante of. 25. men. How many souldiars had he, as you gesse?

The Scholar proceeds by letting x represent the number of soldiers in the front of the battalion, so that x^2 is the number of soldiers in the square battalion. So he finds the number of men to be $x^2 + 284$. When the front is increased by one man, the new formation contains $(x + 1)^2 - 25 = x^2 + 2x - 24$ men. The Scholar observes: "Now haue I one nomber of menne, expressed by. 2 Cossike nombers: Of necessitie therefore must these. 2. nombers be equalle: seyng thei represente one armie." After setting the two expressions equal and subtracting x^2 from both sides, the equation $284 = 2x - 24$ is solved to find $x = 154$. This gives the number of soldiers in the front of the formation. To find the total number of soldiers, this value is substituted for x in the expression $x^2 + 284$ to obtain 24,000 (Recorde, 1557, p. 253-254).

Recorde presents a brain teaser of sorts:

> There is a kyng with a greate armie: And his aduersarie corrupteth one of his heraultes with giftes, and maketh hym swere, that he will tell hym, how many Dukes, Erles and other souldiars there are in that armie. The heraulte lothe to leafe those giftes, and as lothe to be untrue to his Prince, diuiseth his aunswere, which was true, but yet not so plain, that the aduersarie could therby understand that, whiche he desired. And that aunswere was this.
>
> Looke how many Dukes there are, and for eche of them, there are twise so many Erles. And under euery Erle,there are fower tymes so many soldiars, as there be Dukes in the fielde. And when the muster of the soldiars was taken, the. 200. parte of them, was 9. tymes so many as the nomber of the Dukes.
>
> This is a true declaration of eche nomber, quod the heraute: and I haue discharged my othe. Now gesse you how many of eche sorte there was (Recorde, 1557, p. 255).

The Scholar replies, "Although the question seme harde, I see many tymes, that diligence maketh harde thynges easie, and therefore I will attempte the worke of it." He takes the number of dukes to be x, and so the number of earls is $2x^2$. Then the number of soldiers is $8x^3$. Next, because $\frac{1}{200}$ of the number of soldiers is equal to nine times the number of dukes, we have $\frac{1}{200}(8x^3) = 9x$. Multiplying both sides by 200 yields $8x^3 = 1800x$. Then, dividing both sides by $8x$ leads to $x^2 = 225$, and taking the square root gives us $x = 15$. So there are 15 dukes. Substituting 15 for x in $2x^2$ gives us 450 earls, and doing likewise with $8x^3$ gives us 27,000 soldiers (Recorde, 1557, p. 255-256).

Let's investigate one more "question of an armie," as follows:

> One other question I will propounde, of. 2. armies beyng bothe square, and of like nomber. And if you abate. 4. from the one armie, and adde. 10. to the other armie, and then multiplie them bothe together, there will amounte. 9853272. I demaunde of you, what is the fronte of those square battailes.

The Scholar proceeds confidently. He calls the number of soldiers in the front x, so the number of soldiers in each army is x^2. Subtracting 4 from one army, and adding 10 to the other, yields $x^2 - 4$ and $x^2 + 10$, respectively. He then finds the product of these to be $x^4 + 6x^2 - 40$. Setting this equal to 9853272 and completing the square yields $x = 56$ (Recorde, 1557, p. 280-281).

Clearly one can see that *The Whetstone of Witte* is at a more complex level of mathematics than *The Grounde of Artes*, yet it is still representing a mathematics geared towards practical application. The step by step examples presented by Recorde can still serve as a role model for educators. The syntax and words might have changed after more than five hundred years, but the underlying concepts hold and continue to remain sound. It will be interesting to see what mathematical texts will look like five hundred years into the future. Robert Recorde should continue to be a useful historical example for scholars and students in a distant future time.

It becomes all too easy for us of the present, more completely grounded in a scientific paradigm, to look at Recorde and his works as being jejune. Yet, for his time, he was one of only a handful of mathematicians of any real merit (Roberts, 2016, p. 200). Reflecting that his works were still popular well into the age of Newton again shows the weight of his works. Finally, perhaps Recorde's greatest, most lasting, and most modern influence was to make all knowledge available for all to partake of and benefit from(Roberts, 2016, p. 192).

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