

The 72nd Meeting of the  
**Alabama Association of College Teachers of  
Mathematics**

Troy University, Troy, AL  
March 4, 2023

**The 2023 Lewis-Parker Lecture**

*Are These the Real Roots or Just Imaginary? A Historical Journey From Cardano's  
Cubic to New Results.*

**Frank Patane**, Samford University

ABSTRACT

Our talk begins with a historical discussion of solving polynomial equations in one variable. In particular, we examine the quadratic, cubic, and quartic formulas and their place in history. We then look at the symmetry inherent in these roots systems, and the delicate balance between a polynomial's roots and coefficients. This symmetry extends to quintic polynomials, but alas, the existence of a "general formula" does not. The non-existence of a general quintic formula leads us to a discussion of Galois theory and the computation of the Galois group. We then discuss new developments for the computation of the Galois group which only use elementary means that depend only on the polynomial's coefficients. In closing we give the formula for the Galois group of any doubly even octic polynomial and its applications.

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The Alabama Journal of Mathematics is published under the auspices of the Alabama Council of Teachers of Mathematics (ACTM) and the Alabama Association of College Teachers of Mathematics (AACTM). The AJM is designed to meet a number of needs of the mathematics community in the State of Alabama. Specifically, the intent of the Journal is to knit together the various components of this mathematical community. As such, the journal includes research articles in mathematics and mathematics education appropriate for a general audience and activities and problems for K-16 mathematics teachers.

**Acknowledgements**

The AACTM would particularly like to thank Troy University for graciously hosting this year's conference.

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# AACTM Talks: March 4, 2023

CT: Contributed Talk, KL: Keynote Lecture

8:00-8:35	Registration and Refreshments		
8:35-8:45	Welcome remarks		
8:45-9:00	CT	<b>Vitaly Voloshin</b> Troy University	<i>From the History of Mixed Hypergraph Coloring</i>
9:05-9:20	CT	<b>Jaedeok Kim</b> Jacksonville State University	<i>When is <math>n^2 + 1</math> Prime? A New Perspective on Factoring Integers of the Form <math>n^2 + 1</math></i>
9:25-9:40	CT	<b>George Lytle</b> University of Montevallo	<i>Calculus Reinforcement in Real Analysis</i>
9:45-10:00	CT	<b>Andrei Pavelescu</b> University of South Alabama	<i>An Infinite Family of Linklessly Embeddable Tutte-4-Connected Graphs</i>
10:05-10:20	CT	<b>Janie Kennedy</b> Samford University	<i>Precalculus Assessment</i>
10:20-10:40	Break		
10:40-10:55	CT	<b>Pat Rossi</b> Troy University	<i>Grooming Undergraduate Students for Their First Talk - A Retrospective</i>
11:00-12:00	KL	<b>Lewis-Parker Lecture: Frank Patane</b> Samford University	<i>Are These the Real Roots or Just Imaginary? A Historical Journey From Cardano's Cubic to New Results.</i>
12:00-1:00	Lunch and Group Photo		
1:00-1:15	CT	<b>Steven Clontz</b> University of South Alabama	<i>Open-source mathematical infrastructure and student engagement</i>
1:20-1:35	CT	<b>Huijun Yi</b> Troy University	<i>Data mining: Mathematical Perspectives</i>
1:40-1:55	CT	<b>Jim Gleason</b> The University of Alabama	<i>Developing Secondary Teachers' Understanding of Data Analysis</i>
2:00-2:15	CT	<b>Johnathan Herron</b> University of South Alabama	<i>On Minor Minimal Intrinsically Knotted Graphs</i>
2:20-2:35	CT	<b>Danush Wijekularathna</b> Troy University	<i>Effective Mathematics Teaching Strategy</i>
2:40-2:50	Break		
2:50-3:05	CT	<b>Adella Herron</b> University of South Alabama	<i>The Web and Polytabloid Bases</i>
3:10-3:25	CT	<b>Priyojit Palit</b> Spring Hill College	<i>Exploring Quiver Mutations and Dynkin Diagrams for Coxeter and Braid Groups</i>
3:30-3:45	CT	<b>Hoa Dinh</b> Troy University	<i>On Generalized Karcher Mean</i>
3:50	Business Meeting		

# ABSTRACTS (in alphabetical order by speaker's name)

## Open-source mathematical infrastructure and student engagement

*Steven Clontz*

University of South Alabama

Depending on the student and the field of mathematics, it can be quite difficult to find student projects that will result in scholarly contributions that benefit the bleeding edge of mathematics. Likewise, undergraduate students who engage in pure mathematics research are well-prepared for graduate school, but the artifacts of this work often have limited value to industry employers. This presentation will suggest a framework for engaging mathematics students in scholarly projects involving open-source software and skills that prepare students for both advanced mathematical studies and the private sector.

## On Generalized Karcher Mean

*Hoa Dinh*

Troy University

In this talk we introduce a generalized Karcher mean as a unique solution of a matrix equation and as a unique solution of the least squares problem. We also study how to approximately find the generalized Karcher mean.

## Developing Secondary Teachers' Understanding of Data Analysis

*Jim Gleason*

The University of Alabama

With our society's increased dependence upon the use of data and statistical ideas in every aspect of life there is a greater emphasis on statistical topics in the K-12 mathematics course of study. However, many of our pre-service and in-service secondary teachers have not had sufficient preparation for teaching this content because of the lack of statistics courses completed or the different focus of these courses. In this talk, I will give an overview of the statistics contained in the secondary mathematics course of study in Alabama and what we are doing at The University of Alabama to help bridge the gap between teachers' current knowledge and what we are expecting of them.

## The Web and Polytabloid Bases

*Adella Herron*

University of South Alabama

We will discuss two vector spaces: the vector space  $W_n$  generated by the web basis and the vector space  $V_n$  generated by the polytabloid basis. Next, we will delve into the relationship between these two spaces. Specifically, we will focus on the little-understood function which maps from  $W_n$  to  $V_n$ .

## On Minor Minimal Intrinsically Knotted Graphs

*Johnathan Herron*

University of South Alabama

For this presentation, we shall survey the most important definitions and results regarding the topic of intrinsic knotting in graphs, beginning with the basic vocabulary of graph theory and the ideas introduced in J. Conway and C. Gordon's foundational paper "Knots and Links in Spatial Graphs." We shall also consider the relationship between linking and knotting; this includes J. Foisy's lemma regarding the presence of a doubly-linked  $D_4$  graph and how it indicates knotting. These results lead directly to our own research, in which we seek to prove that a graph of thirty edges and ten vertices is minor-minimal intrinsically knotted

## Precalculus Assessment

*Janie Kennedy*

Samford University

Precalculus assessment questions are discussed, including routine multiple choice questions, concept multiple choice questions, and free response questions.

## When is $n^2 + 1$ Prime? A New Perspective on Factoring Integers of the Form $n^2 + 1$

*Jaedeok Kim*

Jacksonville State University

In 1837, Dirichlet proved that infinitely many primes arise in any arithmetic progression  $an + b$  if  $(a, b) = 1$ . Nearly two hundred years have passed since the Dirichlet's famed result, but the infinitude of primes in a quadratic form has yet to be proved. This talk will introduce a method of factoring integers of the form  $n^2 + 1$  by using the continued fraction and Stern-Brocot tree

## Calculus Reinforcement in Real Analysis

*George Lytle*

University of Montevallo

We all know that Real Analysis examines the foundations of calculus, but what good does it do for students to look at foundations of a class they don't remember from three years ago? To address this lack of familiarity, I implemented Calculus Moments in my real analysis course. This systematic review of the calculus core brings students back into the mindset that motivates analysis in the first place. In this talk, we will explore the implementation and success of these Calculus Moments in the Fall 2022 semester.

## Exploring Quiver Mutations and Dynkin Diagrams for Coxeter and Braid Groups

*Priyojit Palit*

Spring Hill College

Coxeter groups and their braid groups can be represented by Dynkin diagrams. In their foundational work on cluster algebras, Fomin and Zelevinsky introduced an operation called mutation on quivers, which are oriented Dynkin diagrams. This raises the question of whether a quiver mutation-equivalent to a Dynkin diagram's orientation also encodes a presentation of a Coxeter or braid group. Barot and Marsh answered this question by constructing such presentations for Coxeter groups through explicit relation writing. Later, Grant and Marsh generalized these results to the corresponding braid groups. We extend these results for simply-laced types using presentations that are encoded by reduced factorizations of a Coxeter element. These generalized results recover the previously mentioned results by specializing to certain twopart factorizations in bijection with vertices of the cluster exchange graph and certain compositions of Hurwitz moves, which parallel quiver mutation

## Are These the Real Roots or Just Imaginary? A Historical Journey From Cardano's Cubic to New Results

*Frank Patane: Lewis-Parker Lecture*

Samford University

Our talk begins with a historical discussion of solving polynomial equations in one variable. In particular, we examine the quadratic, cubic, and quartic formulas and their place in history. We then look at the symmetry inherent in these roots systems, and the delicate balance between a polynomial's roots and coefficients. This symmetry extends to quintic polynomials, but alas, the existence of a "general formula" does not. The non-existence of a general quintic formula leads us to a discussion of Galois theory and the computation of the Galois group. We then discuss new developments for the computation of the Galois group which only use elementary means that depend only on the polynomial's coefficients. In closing we give the formula for the Galois group of any doubly even octic polynomial and its applications.

## An Infinite Family of Linklessly Embeddable Tutte-4-Connected Graphs

*Andrei Pavelescu*

University of South Alabama

In 1997, Maharry produced an example of a simple graph of order 13 which was 4-connected, triangle free (which he called Tutte-4-connected), and linklessly embeddable. It was only the second known such example. Through private communication with Maharry, Robertson had conjectured that  $K_{5,5}$  minus a perfect matching was the only graph with these properties, so Maharry's discovery was quite extraordinary. In this talk, we show that the class of linklessly embeddable Tutte-4-connected graphs is quite rich, as we provide an example of such a graph for every order  $n \geq 14$ .

## Grooming Undergraduate Students for Their First Talk—A Retrospective

*Pat Rossi*

Troy University

The author discusses how to come up with specific topics/ideas for undergraduate talks, examples of student talks given in the past and how students came about giving these talks, strategies for encouraging/helping students who are reluctant to give a talk because of lack of confidence or other obstacles.

## **From the History of Mixed Hypergraph Coloring**

***Vitaly Voloshin***

Troy University

I will talk about the history, people and ideas of mixed hypergraph coloring. More detailed information on the topic can be found on Mixed Hypergraph Coloring website reached from <http://spectrum.troy.edu/voloshin/>.

## **Effective Mathematics Teaching Strategy**

***Danush Wijekularathna***

Troy University

There has been a discussion among educators regarding lesson planning. Although there are numerous formats for lesson planning, the core components and their sequences, such as objectives, homework, and mental mathematics, are fundamental. A teacher's behavior and the behavior of their students are influenced by many decisions he or she makes, both consciously and unconsciously. This study examined implementing the Lesson Planning strategy in planning lessons for instruction. To be successful teachers, we must plan meaningful experiences for our students. Lessons and presentations that are well organized facilitate students' understanding of the relationships between mathematical concepts and the important ideas of mathematics. It is important to recognize that the presentation of information affects how a student constructs new knowledge while learning.

## **Data Mining: Mathematical Perspectives**

***Huijun Yi***

Troy University

One common complaint of students learning mathematics is that the topics covered seem to have little relevance to practical problems. In this talk, I intend to motivate the need for mathematical and statistical concepts in the context of fundamental Data Mining/Machine Learning problems such as classification and cluster analysis.