Allocating U.S. Senate Seats Based on Population: a Hypothetical Exercise

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An important feature of contemporary mathematics teaching involves the integration of statistical activities into the mathematics classroom. For many students, political situations provide a rich source of interesting real world data. We shall present one such example.

The American Congress is composed of two legislative houses. The seats in the House of Representatives are apportioned based upon population, while the Senate has equal representation (two senators) from each state. This arrangement resulted from a famous compromise between the larger and smaller states when the Constitution was framed.

Using the U.S. Constitutional method as a model, most states initially established two legislative houses – one based upon population and one based upon county divisions or other such area-based criteria. However, a series of Supreme Court rulings in the 1960's forced the states to apportion seats in both of their legislative houses based upon population.

Suppose that the U.S. Senate were also to be reconstructed using the "one-person one-vote" principle which is applied to state legislatures. How would each of the 50 states be represented?

Table 1 provides Census Data for each individual state for the years 2000 and 1990.

Table 1:

State	**2000 Census**	**1990 Census**
AL	4,447,100	4,040,587
AK	626,932	550,043
AZ	5,130,632	3,665,228
AR	2,673,400	2,350,725
CA	33,871,648	29,760,021
CO	4,301,261	3,294,394
CT	3,405,565	3,287,116
DE	783,600	666,168
FL	15,982,378	12,937,926
GA	8,186,453	6,478,216
HI	1,211,537	1,108,229
ID	1,293,953	1,006,749
IL	12,419,293	11,430,602
IN	6,080,485	5,544,159
IA	2,926,324	2,776,755
KS	2,688,418	2,477,574
KY	4,041,769	3,685,296
LA	4,468,976	4,219,973
ME	1,274,923	1,227,928
MD	5,296,486	4,781,468
MA	6,349,097	6,016,425
MI	9,938,444	9,295,297
MN	4,919,479	4,375,099
MS	2,844,658	2,573,216
MO	5,595,211	5,117,073

Table 1 (continued):

State	2000 Census	1990 Census
MT	902,195	799,065
NE	1,711,263	1,578,385
NV	1,998,257	1,201,833
NH	1,235,786	1,109,252
NJ	8,414,350	7,730,188
NM	1,819,046	1,515,069
NY	18,976,457	17,990,455
NC	8,049,313	6,628,637
ND	642,200	638,800
OH	11,353,140	10,847,115
OK	3,450,654	3,145,585
OR	3,421,399	2,842,321
PA	12,281,054	11,881,643
RI	1,048,319	1,003,464
SC	4,012,012	3,486,703
SD	754,844	696,004
TN	5,689,283	4,877,185
TX	20,851,820	16,986,510
UT	2,233,169	1,722,850
VT	608,827	562,758
VA	7,078,515	6,187,358
WA	5,894,121	4,866,692
WV	1,808,344	1,793,477
WI	5,363,675	4,891,769
WY	493,782	453,588

To perform this hypothetical apportionment of the U.S. Senate based upon the 2000 census, we can encourage students to work in small groups to complete the following six steps and organize their information based on the 2000 Census Data given in the center column of Table 1. (See the headings for Table 2 as one method that students can use to organize the information. In addition, students can use spread sheets to calculate the tables).

(1) Compute the sum of the populations of the 50 states. (This total is 280,815,006).

(2) Divide this total population by 100, the number of U.S. Senate seats. This quotient (2,808,150.06) represents the "ideal" population for each senator to represent.

(3) Divide each state population by the number obtained in Step 2. This quotient reports for each state the exact number of Senators to which that state should be entitled. We call this the "Exact Senate Quota" (ESQ).

(4) Perform an initial senate seat allocation based upon the ESQ of step 3 in the following way:
   (a) Allocate to each state the same number of Senators as the integer part of their ESQ.
   (b) For those 19 states whose integer ESQ portions are zero, allocate one seat and remove that state from further consideration.

(5) Total the number of senate seats thus far allocated. (This sum is 95).

(6) Arrange in descending rank order the decimal portions of the ESQ for the 31 states which remain in competition for an additional seat. For those 19 states no longer in competition, use 0.000 as the decimal portion. Allocate an additional Senate seat to those five states with the highest decimal rankings. (They are New Jersey, Missouri, Georgia, Wisconsin, and Maryland).

Table 2 reports the results of these six steps. These are the "answers" for the first student assignment.

The same steps can be performed for the 1990 census. Table 3 reports the results of these calculations. These are the "answers" for this second student assignment.

The following questions can be discussed by students individually or in small groups.

(1) Which states gained in hypothetical representation from 1990 to 2000? Which ones lost?

(2) Find census reports for 1980, 1970, 1960, 1950, 1940, ... and construct tables for hypothetical Senate allocations and make comparisons. Write a short paper explaining your conclusions.

(3) Make predictions of state populations in the year 2010 based upon the trends from earlier census data. How might these predictions affect the hypothetical Senate allocation in 2010?

(4) Suppose that apportionment of senators is modified so that states with ESQ's less than 0.5 are paired to provide one senate seat for each pair. How would this affect Tables 2 and 3?

(5) What other methods might be used to apportion the U.S. Senate? Which method is the fairest?

(6) In the 2000 allocation (Table 1) California alone has 12 senators; the combination of only 10 states (CA, TX, NY, FL, IL, PA, OH, NJ, MO, MI) accounts for 50 Senate seats, half of the total. What would be the political effect of this concentration of Senate seats?

(7) For students who have had exposure to the physical sciences, it might also be appropriate to ask the following: Explain why the computations in steps 2 and 3 yield the ESQ - the exact number of senators to which a state should be entitled.

(In Step 2, computing $\frac{\text{U.S. Population}}{\text{100 senators}}$ yields $\frac{\text{people}}{\text{senator}}$, i.e., the number of people per senator. In Step 3, dividing a state's population by the number obtained in Step 2 yields $\frac{\left(\frac{\text{people}}{\text{state}}\right)}{\left(\frac{\text{people}}{\text{senator}}\right)} = \frac{\text{people}}{\text{state}} \cdot \frac{\text{senators}}{\text{person}} = \frac{\text{senators}}{\text{state}}$.)

These questions are a beginning. Mathematics teachers may want to work with colleagues in social studies on joint projects.

Table 2, 2000 Census Data Results:

State	2000 Population	ESQ	Initial Allocation	Adjusted Decimal Portion of ESQ	Additional Allocation	Final Allocation
NJ	8,414,350	2.996	2	0.996	1	3
MO	5,595,211	1.992	1	0.992	1	2
GA	8,186,453	2.915	2	0.915	1	3
WI	5,363,675	1.910	1	0.910	1	2
MD	5,296,486	1.886	1	0.886	1	2
NC	8,049,313	2.866	2	0.866	0	2
AZ	5,130,632	1.827	1	0.827	0	1
NY	18,976,457	6.758	6	0.758	0	6
MN	4,919,479	1.752	1	0.752	0	1
FL	15,982,378	5.691	5	0.691	0	5
LA	4,468,976	1.591	1	0.591	0	1
AL	4,447,100	1.584	1	0.584	0	1
MI	9,938,444	3.539	3	0.539	0	3
CO	4,301,261	1.532	1	0.532	0	1
VA	7,078,515	2.521	2	0.521	0	2
IL	12,419,293	4.443	4	0.443	0	4
KY	4,041,769	1.439	1	0.439	0	1
SC	4,012,012	1.429	1	0.429	0	1
TX	20,851,820	7.425	7	0.425	0	7
PA	12,281,054	4.373	4	0.373	0	4
MA	6,349,097	2.261	2	0.261	0	2
OK	3,450,654	1.229	1	0.229	0	1
OR	3,421,399	1.218	1	0.218	0	1
CT	3,405,565	1.213	1	0.213	0	1
IN	6,080,485	2.165	2	0.165	0	2

Table 2, 2000 Census Data Results (Continued):

State	2000 Population	ESQ	Initial Allocation	Adjusted Decimal Portion of ESQ	Additional Allocation	Final Allocation
WA	5,894,121	2.099	2	0.099	0	2
CA	33,871,648	12.062	12	0.062	0	12
OH	11,353,140	4.043	4	0.043	0	4
IA	2,926,324	1.042	1	0.042	0	1
TN	5,689,283	2.026	2	0.026	0	2
MS	2,844,658	1.013	1	0.013	0	1
AK	626,932	0.223	1	0.000	0	1
AR	2,673,400	0.952	1	0.000	0	1
DE	783,600	0.279	1	0.000	0	1
HI	1,211,537	0.431	1	0.000	0	1
ID	1,293,953	0.461	1	0.000	0	1
KS	2,688,418	0.957	1	0.000	0	1
ME	1,274,923	0.454	1	0.000	0	1
MT	902,195	0.321	1	0.000	0	1
NE	1,711,263	0.609	1	0.000	0	1
NV	1,998,257	0.712	1	0.000	0	1
NH	1,235,786	0.440	1	0.000	0	1
NM	1,819,046	0.648	1	0.000	0	1
ND	642,200	0.229	1	0.000	0	1
RI	1,048,319	0.373	1	0.000	0	1
SD	754,844	0.269	1	0.000	0	1
UT	2,233,169	0.795	1	0.000	0	1
VT	608,827	0.217	1	0.000	0	1
WV	1,808,344	0.644	1	0.000	0	1
WY	493,782	0.176	1	0.000	0	1
Total	280,815,006		95		5	100

Table 3, 1990 Census Data Results:

State	2000 Population	ESQ	Initial Allocation	Adjusted Decimal Portion of ESQ	Additional Allocation	Final Allocation
CA	29,760,021	11.995	11	0.995	1	12
WI	4,891,769	1.972	1	0.972	1	2
TN	4,877,185	1.966	1	0.966	1	2
WA	4,866,692	1.962	1	0.962	1	2
MD	4,781,468	1.927	1	0.927	1	2
TX	16,986,510	6.847	6	0.847	1	7
PA	11,881,643	4.789	4	0.789	0	4
MN	4,375,099	1.763	1	0.763	0	1
MI	9,295,297	3.747	3	0.747	0	3
LA	4,219,973	1.701	1	0.701	0	1
NC	6,628,637	2.672	2	0.672	0	2
AL	4,040,587	1.629	1	0.629	0	1
GA	6,478,216	2.611	2	0.611	0	2
IL	11,460,602	4.607	4	0.607	0	4
VA	6,187,358	2.494	2	0.494	0	2
KY	3,685,296	1.485	1	0.485	0	1
AZ	3,665,228	1.477	1	0.477	0	1
MA	6,016,425	2.425	2	0.425	0	2
SC	3,486,703	1.405	1	0.405	0	1
OH	10,847,115	4.372	4	0.372	0	4
CO	3,294,394	1.328	1	0.328	0	1
CT	3,287,116	1.325	1	0.325	0	1
OK	3,145,585	1.268	1	0.268	0	1
NY	17,990,455	7.251	7	0.251	0	7
IN	5,544,159	2.235	2	0.235	0	2

Table 3, 1990 Census Data Results (Continued):

State	1990 Population	ESQ	Initial Allocation	Adjusted Decimal Portion of ESQ	Additional Allocation	Final Allocation
FL	12,937,926	5.215	5	0.215	0	5
OR	2,842,321	1.146	1	0.146	0	1
IA	2,776,755	1.119	1	0.119	0	1
NJ	7,730,188	3.116	3	0.116	0	3
MO	5,117,073	2.062	2	0.062	0	2
MS	2,573,216	1.037	1	0.037	0	1
AK	550,043	0.222	1	0.000	0	1
AR	2,350,725	0.947	1	0.000	0	1
DE	666,168	0.269	1	0.000	0	1
HI	1,108,229	0.447	1	0.000	0	1
ID	1,006,749	0.406	1	0.000	0	1
KS	2,477,574	0.999	1	0.000	0	1
ME	1,227,928	0.495	1	0.000	0	1
MT	799,065	0.322	1	0.000	0	1
NE	1,578,385	0.636	1	0.000	0	1
NV	1,201,833	0.484	1	0.000	0	1
NH	1,109,252	0.447	1	0.000	0	1
NM	1,515,069	0.611	1	0.000	0	1
ND	638,800	0.257	1	0.000	0	1
RI	1,003,464	0.404	1	0.000	0	1
SD	696,004	0.281	1	0.000	0	1
UT	1,722,850	0.694	1	0.000	0	1
VT	562,758	0.227	1	0.000	0	1
WV	1,793,477	0.723	1	0.000	0	1
WY	453,588	0.183	1	0.000	0	1
Total	248,102,973		94		6	100

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