Problems

(1) Find two square matrices A and B such that $AB = 0$ but $BA \neq 0$. Here 0 denotes a matrix with all zero entries.

(2) If $w+x+y+z = 12$ and $w^2+x^2+y^2+z^2 = 48$, determine the largest possible value of w.

(3) Suppose that N atoms are moving to the right at unit speed within the unit interval $[0, 1]$. Whenever an atom reaches either endpoint 0 or 1, or collides with another atom, it immediately bounces and moves at the same speed in the opposite direction. This continues until all atoms return to their original positions while moving to the right. Count the total number of times that two atoms collide. (Hint: first consider small values of N, and look for a pattern.)

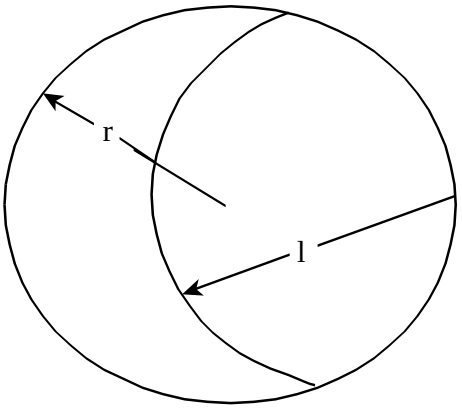
(4) A sack contains marbles numbered $1, 2, \ldots, N$. Randomly draw three marbles, with replacement after each draw. Determine the probability that the sum of the numbers on the three drawn marbles is divisible by 3. (Note: N might not be a multiple of 3.)

(5) Suppose that C_1, C_2, and C_3 are non-overlapping circles with distinct radii drawn in the plane. For $i \neq j$, let P_{ij} denote the intersection point of the two external tangents common to C_i and C_j. Prove that points P_{12}, P_{13}, and P_{23} are collinear.

(6) Prove that $2e^x > 3x^2$ for all $x \geq 0$.

(7) Chose three digits a, b, c such that $9 \geq a > b > c \geq 1$. Compute the difference $abc - cba$, and call the result xyz. Prove that the sum of xyz and zyx is always 1089.

(8) A farmer has a circular pasture of radius r, and sheep which grazes in the pasture. The farmer intends to chain the sheep to a hitching post on the circumference of the circle. What should the length l of the chain be in order for the sheep to graze on exactly half of the pasture?



Solutions, comments, and discussions should be sent to:

Ken Roblee
Department of Math & Physics
232 MSCX
Troy State University
Troy, AL 36082
(334)670-3406
FAX (334)670-3796
kroblee@troy.edu

Vicky Eichelberg
Saint James School
6010 Vaughn Road
Montgomery, AL 36116
(334)277-8033
FAX (334)277-8059
eichelberg2@charter.net