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Activities

Trigonometry in Context

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Introduction

Applications of trigonometry are often of little interest to students when they are presented in contrived scenarios or abstracted by removing the context. One way to capture students' imagination and interest in trigonometry is to use photographs of local buildings, objects, designs, or art as a context for problem solving. Since August 2004, the *Mathematics Teacher* has included a department called *Mathematics Lens* in which photographs serve as the basis for interesting mathematics problems. The use of photographs gives a feeling of authenticity to the problem, it captures a sense of reality, and it provides students some insight into the value of mathematics.

Three activities are included below in which a photograph is presented and mathematical questions are posed. Having participated in such activities, students can then be given the assignment of using digital cameras to take their own pictures of objects or situations in which they see mathematics, formulating their own mathematics problems related to the photographs, providing solutions to the problems, and then compiling all of the above as a project. I have found that when students work in pairs on this activity, they create more interesting and mathematically sophisticated problems. The activity provides a window into the type of mathematics that students see in their world and how the mathematics that they are learning relates to their surroundings.

Activities

Activity 1: Decorating a Gateway

Photo 1

Photo 1 shows a picture of a security barrier placed above a gate at St. John's College, Johannesburg, South Africa. The vertical bars in the semi-circle are evenly spaced with a gap of 12 cm between successive bars. The exterior diameter of circles A and B is also 12cm. The center of circle A is vertically above the first vertical bar inside the inner semi-circle. The centre of circle B is vertically above the right edge of the inner semi-circle.

How far apart on the semi-circle are the points of tangency of circles A and B to the semi-circle? (The situation is illustrated in Figure 1 below.)

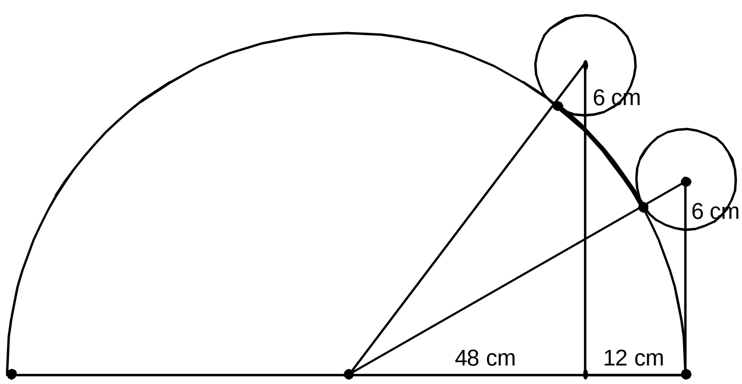
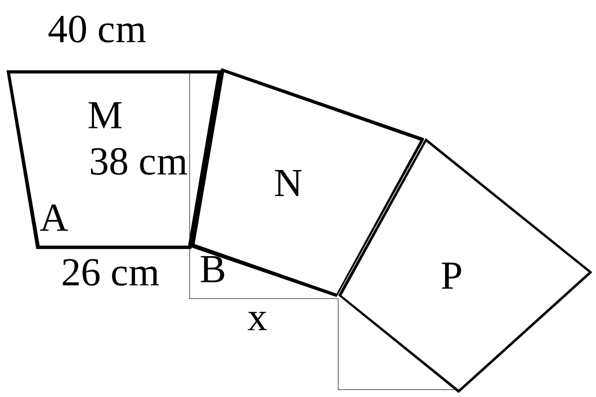


Figure 1

Activity 2: The Span of an Arch Bridge

Picture 2

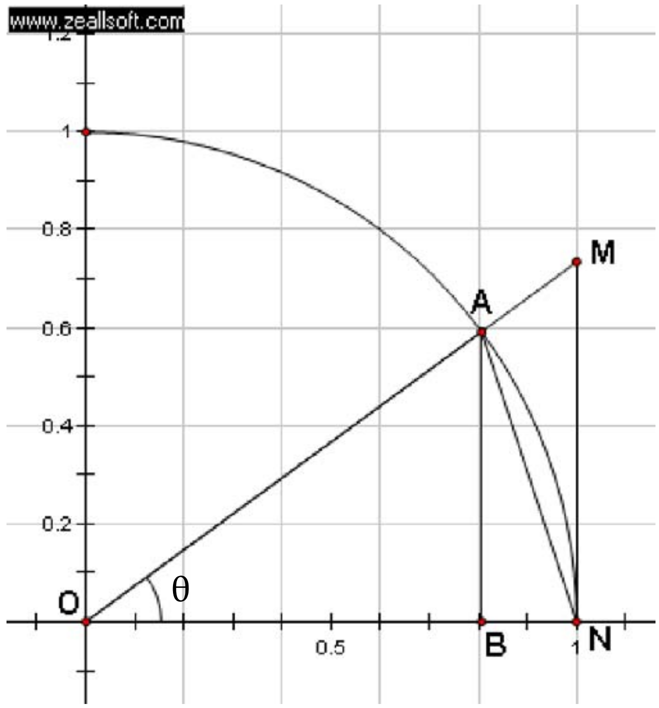
In the SciBono Exploration Centre, Johannesburg, South Africa, there is a model of an arch bridge built from isosceles trapezoids and held together by nothing but the central keystone. This model is a wonderful example of how an arch bridge transfers the downward force exerted on the bridge outwards in a horizontal direction.



a) Determine the size of angle A and hence the size of angle B.
b) Determine the horizontal distance x covered by trapezoid N
c) Determine the span, RS, of the bridge

Activity 3: The Pinching Theorem

To develop the Calculus of trigonometric functions, an important theorem in trigonometry is needed. The investigation and subsequent proof of this theorem is very elegant and does a great job of showcasing the power and simplicity of mathematics.



a) Determine the area of ΔAON as a (trigonometric) function of θ.
b) Determine the area of sector AON in terms of θ.
c) Determine the area of ΔMON as a (trigonometric) function of θ.
d) Set up an inequality representing the relationship between the areas of ΔAON, sector AON, and ΔMON.

$\_\_\_\_\_\_\_\_\_\_ \leq \_\_\_\_\_\_\_\_\_\_ \leq \_\_\_\_\_\_\_\_\_\_$

e) Simplify this inequality by dividing all three statements by $\frac{1}{2}\sin(\theta)$.
f) As the measure of angle θ gets closer to zero, determine the value that $\frac{\theta}{\sin(\theta)}$ approaches.
g) Hence, as $\theta \to 0$, $\frac{\sin(\theta)}{\theta} \to$ __________.

Teacher Notes

Activity 1: Decorating a Gateway

The problem solving and insight required to set up the trigonometry in this problem is a good challenge for students in grades 11 and 12. The sketch diagram helps students make sense of the question and gives them an anchor for solving the problem. If you want a more challenging question for your students remove the sketch diagram from the activity sheet.

Solution

Let θ_A and θ_B be the angles between the horizontal and the radii of the semi-circle at the points of tangency to circles A and B, respectively.

$\cos(\theta_A) = \frac{48}{66} = 0.7565$ radians

$\cos(\theta_B) = \frac{60}{66} = 0.4297$ radians

Arc length $= \theta_r = (0.7565 - 0.4297) \cdot 60 = 19.61$ cm

Activity 2: The Span of an Arch Bridge

This photo of an arch bridge with a trapezoidal keystone was taken at a local science exploration center. Similar contexts for photographs can include a long pendulum, a Ferris wheel, a theodolite (survey instrument) or even a WW II artillery tables. As in activity 1, if you want to create a more challenging question for your students, remove the sketch diagram from the activity sheet.

Solution

a) $A = 90° + \tan^{-1}(\frac{7}{38}) = 100.4°$

$B = 360° - 90° - 100.4° - 100.4° = 69.2°$

b) $x = 26 \cdot \sin(69.2°) = 24.3$ cm

c) The second trapezoid horizontal distance $= 26 \cdot \sin(48.4°) = 19.44$ cm

$RS = 26 + 2(24.3) + 2(19.44) = 113.48$ cm

Activity 3: The Pinching Theorem

Solving interesting trigonometry problems in context would not be complete without an elegant problem in the wonderful context of mathematics itself. The "pinching theorem" is one of the more elegant theorems in mathematics - so simple and yet so insightful. Anyone with a basic understanding of trigonometry and radian measure can work through the proof of the theorem. Eudoxus formulated the idea of pinching or squeezing unknown shapes or quantities between two known shapes. His methods allowed Archimedes to approximate π to a level of accuracy that was quite phenomenal for the time. The development and proof of the pinching theorem is a result ascribed to none other than Gauss. Interestingly, in Russia, the pinching theorem is also known as the two militsioner theorem. This name is derived from a story that is told in conjunction with the theorem. If two police officers are holding a prisoner between them, and both of the officers are walking to the prison, the prisoner must also be walking to prison (Wikipedia, 2007).

Solution

a) Area of $\Delta AON = \frac{1}{2}\sin(\theta)$.
b) Area of sector $AON = \frac{1}{2}\theta$.
c) Area of $\Delta MON = \frac{1}{2}\tan(\theta)$.
d) $\frac{1}{2}\sin(\theta) \leq \frac{1}{2}\theta \leq \frac{1}{2}\tan(\theta)$.
e) $1 \leq \frac{\theta}{\sin(\theta)} \leq \frac{1}{\cos(\theta)}$.
f) $\frac{\theta}{\sin(\theta)}$ approaches 1.
g) $\frac{\theta}{\sin(\theta)} \to 1$.

References

1. Wikipedia (downloaded 30 June 2007)
http://en.wikipedia.org/wiki/Squeeze_theorem

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