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Geometer's Sketchpad Discovery Lessons

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Introduction

The Geometer's Sketchpad is a mathematics software program that allows teachers and students to explore geometry concepts and topics. The program enables users to draw, manipulate, and measure any constructions made. Sketchpad allows students to take an active role in their learning rather than the traditional passive role. Students are actively making their own choices, generating and investigating information, making design decisions, and evaluating their own progress. The program allows students to focus more on mathematical concepts and theory by allowing easy manipulations and calculations. Sketchpad enables students to make the connection between concrete mathematics and abstract mathematics.

The following activities utilize *The Geometer's Sketchpad* and are designed as discovery lessons in which the students investigate a problem and develop and test conjectures related to the problem. Using Sketchpad, the students are able to direct their own learning and create their own theories regarding the geometric ideas.

In this paper, we include two lessons which illustrate the utility and versatility of Sketchpad.

Ella and Jo's Land Dilemma

Materials: Computer Lab
The Geometer's Sketchpad computer software

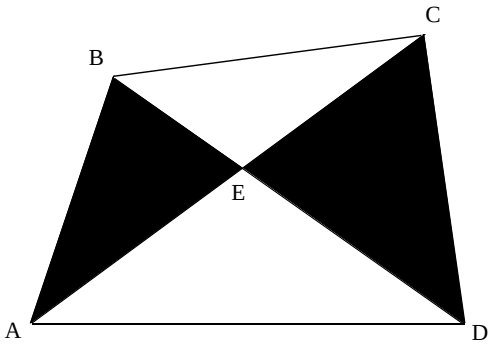
Procedure:

Give the following problem to students to read. Then have students investigate the problem by answering the following questions. Have students share their findings with other classmates.

Problem:

When farmers Clarence and Myrtle died, they left two daughters their land with instructions to divide it equally. One daughter, Ella, was considerably brighter (and more conniving) than her sister, Jo. The land, unfortunately, was shaped as an irregular quadrilateral, and it wasn't immediately obvious how to divide it equally. Ella first tried to get Jo to agree to split it down the diagonal AC shown, with Ella getting region ACD and Jo getting region ABC. Even Jo could see that was a bad deal, so she called her lawyer.

Ella then offered to spit the land with both diagonals. Ella would take two regions: AED and BEC, leaving Jo with ABE and CED. This sounded good to Jo, but her lawyer checked it out and reported that the sums of the areas of the respective regions were still not equal. "Ah," said Ella, "but the products of the areas of our regions *are* equal!" This stumped Jo and her lawyer and she agreed to the deal out of sheer awe for Ella's discovery.



Investigate:

(1) Do you believe Ella's conjecture is true or false? Why?
(2) Test Ella's conjecture by modeling this problem in Sketchpad. Construct an irregular quadrilateral similar to the one above. Label points exactly as in diagram.
(3) Calculate the areas of each of the triangular regions, the sums of the areas of Ella and Jo's regions, and the products of the areas of Ella and Jo's regions using Sketchpad. What did you discover?
(4) Is Ella's conjecture true or false?
(5) Does this mean that this was a fair way to divide the land? Why or why not?
(6) Suppose you are Jo's lawyer. What do you think will be the best way to divide the land up fairly for Ella and Jo? Before you do this construction, define what your definition of fair is in this particular case. How will you determine what is fair for both girls?
(7) Construct your model in Sketchpad. What did you construct in your model?
(8) What did you measure in your model?
(9) Do you feel confident that your measurements support the proposed solution to the problem?
(10) Do you feel your measurements support your definition of "fair" for Ella and Jo?

Castle Builder

Materials: Computer Lab
The Geometer's Sketchpad computer software

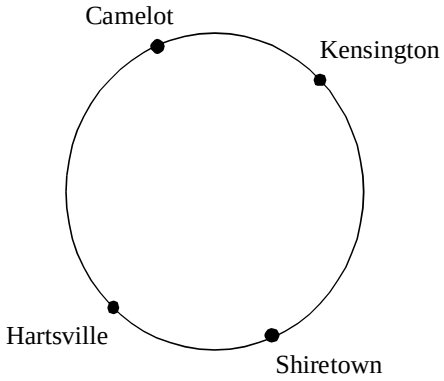
Procedure:

Give the following problem to students to read. Then have students investigate the problem by answering the following questions.

Problem:

You are the king or queen of a new kingdom and you are deciding where you would like to build your castle. You decide that you want to place your castle so that it is a fair traveling distance for the messengers from each of the four major towns. Fortunately,

the four major towns: Camelot, Kensington, Hartsville, and Shiretown, all form a circle with their locations. Where should you build your castle so as to be fair to the messengers from all four towns?



Investigate:

(1) Before you decide where to place your castle, define what your definition of "fair" is in this particular case. That is, how will you determine what is fair for all four towns?
(2) Where are you going to build your castle?
(3) Why have you chosen this particular location? Use mathematical terms and concepts to support your decision.
(4) Model this problem in Sketchpad by constructing a circle similar to the one above. Label the towns exactly as in the diagram. Construct and label your castle.
(5) Where did you place your castle? How did you go about finding this location? Did you randomly place the castle or did you use some mathematical concepts and terms to find the location?
(6) Is your castle's location fair to all four towns? That is, does your castle's location support your definition of fair for each of the four towns?
(7) Can you support your solution with measurements? What are you measuring and why? Do you feel confident that your measurements support the proposed solution to the problem?
(8) Define the following terms related to a circle: center, radius, chord, midpoint, perpendicular bisector.
(9) Construct another circle modeling the above problem with towns appropriately labeled.

(10) In your circle, construct two chords using the towns. Construct the midpoints of the two chords. Construct the perpendicular bisectors of the two chords.
(11) Where do they intersect? How do we know? What can we measure to support our answer? Do you feel that these measurements support your original conjecture of where to place the castle?
(12) Do they (the perpendicular bisectors) always intersect in the same place? How do we know? Does your sketch provide enough of a proof for the solution?
(13) Is this a fair location to place the castle? Why?

References

(1) National Science Foundation, (1993). *Exploring Geometry With The Geometer's Sketchpad: Dynamic Geometry For The 21st Century [Computer software].* Emeryville, CA: Key Curriculum Press.