Problems

Problem 1.

Compute $10^{1024} \bmod 23$ without using a calculator or a computer. [Hint: use repeated squaring.]

Problem 2.

Suppose you have one large chocolate bar that consists of 24 small squares arranged in the shape of a 4-by-6 matrix. Determine the minimum possible number of times you will need to break the bar so that each of the 24 small squares is separated from all the others. You may only break one bar at a time, and you may only break each bar along a straight line. Also try to generalize your answer by considering a chocolate bar that consists of xy small squares arranged in the shape of an x-by-y matrix.

Problem 3.

Start with a cube whose edges have length 1. Draw a circle with diameter 1 on each of this cube's 6 faces. Any 2 circles on opposite faces can be connected to form a cylinder, and there are 3 such cylinders. Find the volume of the region that lies inside the cube but outside all 3 cylinders.

Problem 4.

Evaluate each of the following limits. [Note: you can grind through the messy details, or you can apply general concepts to solve these in your head.]

(1) (a) $\lim_{n\to\infty} \frac{n^2-9999n-99999999}{999999999+9999n+n^2}$

(b) $\lim_{n\to\infty} \frac{2^n}{\log n}$

(c) $\lim_{n\to\infty} \frac{\log^3 n}{2^{2^n}}$

Problem 5.

A simple game begins with 11 stones arranged in a single pile. Two players take alternating turns. Each turn consists of selecting any pile that contains at least 3 stones, and then splitting this pile into two smaller piles. The only restriction is that, after each turn, all the currently remaining piles must contain different numbers of stones. The game ends when one of the players can make no legal move, and this player is declared to be the loser. Assuming that both players want to win the game, what should be the strategy of the first player on his/her first turn?

Solutions, comments and discussions should be sent, no later than November 15, 2002, to:

Mike Hudak or Pat Rossi
Dept. of Math & Physics
232 MSCX
Troy State University
Troy, AL 36082
(334)670-3406
FAX (334)670-3796
mhudak@trojan.troyst.edu
prossi@trojan.troyst.edu

Vicky Eichelberg
Saint James School
6010 Vaughn Road
Montgomery, AL 36116
(334)277-8033
FAX (334)277-8059
eichelberg2@charter.net