Book Review

The Mathematical Olympiad Handbook, by A. Gardiner, Oxford University Press, 1997.

Problem Solving Strategies, by Arthur Engel, Springer-Verlag, 1998.

The Art and Craft of Problem Solving, by Paul Zeitz, John Wiley & Sons Inc., 1999.

Mathematical Olympiad Challenges, by Titu Andreescu and Răzvan Gelca, Birkhäuser, 2000.

Reviewed By Jeff Dodd

In the first pages of *The Art and Craft of Problem Solving,* Paul Zeitz highlights an important distinction between two types of mathematical activities: exercises and problems. As a fervent believer in the precise use of language, I realized, as soon as I thought about it, that many of us tend to use these words almost interchangeably, though we should really be more careful. Exercises are designed to yield to the application of certain prepackaged techniques and procedures; it is (or should be) clear what the path is to the solution of an exercise. The path may be long and difficult, but it has already been demarcated and cleared. Problems, on the other hand, are stated in such a way as to suggest no particular method of solution. They require the solver to combine facts and ideas in novel ways. They require real trial and error investigation, similar, in some respects, to mathematical research.

To what extent can problem solving be taught? Of course, some of us are natively more clever than others, but we can all learn how to be better problem solvers and, as a result, better mathematicians. This is the belief of the authors; all of whom are renowned problem posers, problem solvers, and coaches, who have achieved distinction in all these capacities at the highest levels of international mathematics competition.

I ended up purchasing these four books recently, after looking around quite a bit for books providing coherent and thoughtful introductions to problem solving. There are many other fine problem-solving books on the market (many of which are referenced in these four), but these attracted me because each is intended to share the author's knowledge and experience in an organized fashion. Each is a thorough tutorial, presenting not only general problem solving heuristics, but also many details on specific mathematical ideas and their applications to problem solving. In particular, background material is included on the mathematics that is used, highlighting many useful elementary ideas which do not receive the attention they deserve in standard high school and beginning college courses. Each contains a wealth of problems.

Of the four, the one most explicitly constructed for beginners and based on the most elementary mathematics, seems to be *The Mathematical Olympiad Handbook.* The problems are taken from the first thirty-two British Mathematical Olympiads (1965 - 1996) and are given chronologically. Detailed outlines of solutions are given for all the problems, many in a "fill in the blank" style. The material is "calculus-free." A particularly large set of references is given for other books and sources of problems, many of which, are not widely known or circulated.

The other three books follow an exposition-examples-problems format, with problems clustered, by theme, according to the background material presented. Of these, the one set up most like a textbook is *The Art and Craft of Problem Solving.* In fact, it seems to be written primarily as a textbook for a problem-solving course aimed at beginning college students. Each section contains a generous and leisurely exposition and many worked examples. Complete solutions are not given for all the problems (only hints for some), but to those who adopt the book for a course, help is available in the form of the Instructor's Resource Manual and the Student Solutions Manual. This book has more on general problem solving heuristics than the others, but, being oriented towards a college audience, has almost no geometry. The last of the eight chapters is devoted to calculus based material.

Mathematical Olympiad Challenges would also be suitable for a textbook, though it would probably require a more select group of students. It seems to be written primarily to prepare students for participation in serious Olympiad competition. The exposition is brief, and just one or two examples are worked out before each problem set, but very detailed solutions are given for all the problems. Problems are arranged in order of increasing difficulty. Calculus makes an appearance here and there but is rarely indispensable. On the other hand, vectors and matrices do appear quite a bit.

Problem Solving Strategies is the thickest of the four books, containing more than 1300 examples and problems. Each section contains an exposition of principles, which is brief but absolutely packed with interesting and useful facts, followed by (or interwoven with) a large number of worked examples and a very large number of problems. It may very well be the most comprehensive book of its kind on the market, and, as such, is a valuable reference book for any mathematician. While it has something for everyone, it might be intimidating, at first glance, for those with less mathematical experience and maturity. The problems are arranged in no particular order; but hints, outlines of solutions, or complete solutions are given for virtually all of the problems. Calculus, vectors, and matrices are used in some of the sections.

We would all like to have an experienced coach at our side to help us learn about problem solving. Perhaps owning one of these books is the next best thing.

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