Problems

(1) For each $D \geq 2$, determine all D-digit numbers N such that taking the last D digits of N^2 yields N.

(2) In the tiny nation of Piconesia, currency only exists in denominations of 8 picons and 13 picons. Determine the largest integer number of picons that Piconesians cannot pay exactly without receiving change. Also, in any nation whose currency only has denominations of M and N units, determine the largest integer amount that cannot be paid without receiving change. (Assume that M and N are relatively prime.)

(3) Construct a function $F(x_1, y_1, x_2, y_2, x_3, y_3)$ such that $F(x_1, y_1, x_2, y_2, x_3, y_3) = 0$, if and only if points (x_1, y_1), (x_2, y_2), and (x_3, y_3) form the corners of an equilateral triangle.

(4) Let C denote a circle with radius R. Let $WXYZ$ be a rectangle such that points W and Y lie on circle C, point X is in the interior of C, and point Z is exterior to C. Determine the maximum possible distance from Z to circle C.

(5) Define set S as follows:

$$S = \{n \in \mathbf{N} : \ n \text{ has no prime factor larger than } 11\}.$$

Compute the sum of the reciprocals of all the values in set S.

(6) Find all integer pairs (x, y) such that $x^3 + y^3 = 6xy$.

Solutions, comments, and discussions should be sent to:

Ken Roblee
Department of Math & Physics
232 MSCX
Troy State University
Troy, AL 36082
(334)670-3406
FAX (334)670-3796
kroblee@troyst.edu

Vicky Eichelberg
Saint James School
6010 Vaughn Road
Montgomery, AL 36116
(334)277-8033
FAX (334)277-8059
eichelberg2@charter.net