Solutions and Discussions

Problem 7 — Volume 31, No. 1, Spring, 2007

Chose three digits a, b, c such that $9 \geq a > b > c \geq 1$. Compute the difference $abc - cba$, and call the result xyz. Prove that the sum of xyz and zyx is always 1089.

Solution

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We will solve a more general problem. Specifically, we will assume that A, B, C are digits in base n, where $n > 2$, and $(n-1) \geq A > B > C \geq 0$. Forming the three-digit numbers ABC and CBA (in base n), and letting x, y, z be digits in base n such that the three-digit, base n number xyz is the difference of ABC and CBA,

$$\text{i.e.,} \quad \begin{array}{rccc} & A & B & C \\ - & C & B & A \\ \hline & x & y & z \end{array},$$

we will show that

$$\begin{array}{rcccc} & & x & y & z \\ + & & z & y & x \\ \hline & 1 & 0 & (n-2) & (n-1) \end{array}.$$

Since we perform the operation of subtraction from right to left, we will determine the value of z, then y, and then x.

$\boxed{z = C - A}$ Note that $A > C$. When subtracting a larger digit from a smaller digit, we "borrow" from the digit to the left (B), turning the number ABC into $A\,(B-1)\,(n+C)$, by slight abuse of place value notation. Thus,

$$z = n + C - A.$$

$\boxed{y = (B-1) - B}$ Again, we "borrow" from A, turning ABC into $(A-1)\,(n+B-1)\,(n+C)$. Thus,

$$y = (n + B - 1) - B = n - 1.$$

$$\text{i.e., } y = n - 1.$$

This leaves

$$x = A - 1 + C.$$

Thus, we have:

$$xyz = \underbrace{(A-1+C)}_{x}\underbrace{(n-1)}_{y}\underbrace{(n+C-A)}_{z}.$$

$$\begin{array}{lccccccc} \textbf{Note:} & xyz & & x \cdot n^2 & + & y \cdot n^1 & + & z \cdot n^0. \\ \textbf{Also Note:} & zyx & = & z \cdot n^2 & + & y \cdot n^1 & + & x \cdot n^0. \end{array}$$

Observe:

$$\begin{aligned} xyz + zyx &= (x+z) \cdot n^2 + (2y) \cdot n^1 + (x+z) \cdot n^0 \\ &= (n-1) \cdot n^2 + (2n-2) \cdot n^1 + (n-1) \cdot n^0 \\ &= n \cdot n^2 + (n-2) \cdot n^1 + (n-1) \cdot n^0 \\ &= 1 \cdot n^3 + 0 \cdot n^2 + (n-2) \cdot n^1 + (n-1) \cdot n^0 \end{aligned}$$

$$\text{i.e.,} \quad \begin{array}{ccccc} & & x & y & z \\ + & & z & y & x \\ \hline & 1 & 0 & (n-2) & (n-1) \end{array}.$$

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Extended 90 minute hands-on sessions for each grade band and 60 minute informative sessions for each grade band on various topics that affect teaching mathematics

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