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The Search for Significance: Accuracy in Measurement and Computation

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Introduction

Contrary to what most people think, it is impossible to measure anything with 100% accuracy; there is always some degree of uncertainty. Measurements in science are sometimes reported with a $\pm$ amount, the "give or take" of the measurement. Using such measurements in calculations compounds the uncertainty and can alter the final result enough to cause problems in the real worlds of medicine, engineering and manufacturing, business, etc. The use of significant figures is an attempt to incorporate error analysis for calculations, where the exact standard deviations of measurements are not known, by using digits that are statistically significant.

In science, the way in which the proper number of significant figures is determined is based upon whether something is being measured or whether measurements are being used in calculations. As a rule, with measurements, the last significant digit is the first estimated, and, thus, the first uncertain digit. The appropriate number of significant digits for a calculated value is governed by conventional rules whose applications are dependent on the specific mathematical operations involved. This article looks at both cases through an activity designed to help students identify significant digits, use them properly, and understand their importance in every day life.

Materials:
* A variety of measuring tools. Make sure to have enough of the ones the students will be able to use but include others, such as a tire pressure gauge, spring scale, thermometer, etc., as discussion items
* Enough of the same rectangular-prism-shaped and cylindrical items for all groups
* Two copies of the worksheet for each person in the group
* The significant digit rules handout Calculators

Handouts:
* Accuracy and Significant Digits in Measurement (Student Worksheet)
* Accuracy and Significant Digit Rules

Activity

Divide students into groups of two or three. Provide each group with one worksheet and the same items to measure, but let the students decide from the available measurement tools which one(s) to use, the units, and how many numbers to record for their measurements and calculations. When the students have completed their measurements and calculations and have had an opportunity to compare their results with at least one other group, ask the students to share their findings and answers to the questions on the worksheet. Discuss possible reasons for differences in answers (precision of measuring instrument, human error in measuring, rounding differently, using different units, etc.) From the answers to class discussion (see questions below), have the class develop "rules" to insure accurate, uniform, and consistently reproducible results in both measurement and computation. Write the rules on the board or overhead.

Introduce the scientific concepts of accuracy and precision. Accuracy refers to how close the measured value is to the actual or true value of the object being measured. No measurement is totally accurate due to instrument error or human error, hence the need for some convenient way—such as significant digits—to know by simply looking at a measurement just how accurate it is. On the other hand, precision in the scientific world refers to the ability to reproduce the measurements, accurate or not. Thus, it is possible, for measurements to be precise without being accurate. The classic example of noting the differences between accuracy and precision is the "arrow and target". Accuracy is the idea of where arrows land on a bull's-eye target; they are accurate when they hit the bull's-eye. They are precise if all the arrows cluster close together, even if that is far away from the bull's-eye.

Significant digits are shorthand statistical numbers that indicate the accuracy of the measurement. The use of significant digits is governed by a body of conventions or rules, one set for measured values and another for computed values. Distribute to the class and discuss the significant digit rules handout. Have students compare their rules with the ones on the handout. Guide them in using the rules on selected computations; then, have them complete some on their own. (You might wish to ask a chemistry or physics teacher for worksheets on using significant digits in science activities and computations.)

Have the class decide which units to use in measuring the activity items; give groups a second worksheet and have them repeat the activity using their new understanding of accuracy and significant figures. Tell them that, this time, their results must match those of at least two other groups to show the results are reproducible. Compare all groups' answers using a chart on the overhead or board to facilitate additional class discussion.

Discussion Topics

(1) What are some things that can be measured? Make sure to include things such as time, temperature, air pressure—things other than length and width.
(2) Name as many different measuring instruments as possible and describe what they measure and how they measure it.
(3) What is the difference between accuracy and precision?
(4) Are accuracy and/or precision important? Why, or why not?
(5) What are rules for scientific rounding?
(6) Why is the use of significant digits important in science and mathematics? Where else would the use of significant digits be important?

Follow-up Activities

(1) Assign real-world problems requiring the use of significant digit and rounding rules
(2) Do a science lab requiring accuracy in measurements and the reporting of results
(3) Simulate a research project. Assign groups to be independent research groups that are seeking to replicate, and thus verify, a previous study.

Accuracy and Significant Digits in Measurement

Name ______________________ Date _____________

Instructions: With a partner, take the measurements shown for the item listed and perform the indicated calculations. Be sure to include units in all cases. Then answer the questions that follow. Use an additional sheet to answer the questions if you need more room.

Item	Rectangular Prism	Cylinder
Length		
Width		
Height		
Surface Area		
Volume		

Questions:

1. How did you decide which measuring tool to use? Are some tools better than others for making the required measurements? Explain.

2. How did you decide what units to use?

3. Did you round to get your measurements / answers? If so, how did you decide what number to write down?

4. Compare your measurements and calculations with at least two other groups. Are they the same? Why or why not?

5. Getting the same answer every time no matter who does the measuring or calculating is important in the real world. Can you suggest ways that everyone could routinely get the same answer?

The scientific concept of accuracy along with the use of significant digit rules is one way to assure that measurements and calculations are reproducible. Read the handout about accuracy and significant digits. Then repeat this activity.

Accuracy and Significant Digit Rules

Accuracy in measurement refers to the relationship between the measured value and the true value of what is measured. Contrary to what most people think, it is impossible to measure anything with 100% accuracy; there is always some degree of uncertainty. Measurements in science are sometimes reported with a $\pm$ amount, the "give or take of the measurement. Using such measurements in calculations compounds the uncertainty and can alter the final result enough to cause problems in the real world areas such as medicine, engineering and manufacturing, and business.

Significant figures are an attempt to incorporate error analysis for calculations where the exact standard deviations of measurements are not known by using digits that are statistically significant. In science, the way in which the proper number of significant figures is determined is based upon whether something is being measured or whether measurements are being used in calculations. The last significant digit in a *measured* value is the first estimated position; in other words, the first estimated position is the last significant digit reported in the measured value. The proper number of significant digits for a *computed* value is decided by a set of conventional rules.

Rules for Determining the Number of Significant Digits in Numbers Used in Computations

A. Numbers with Decimals

The decimal tells to what position of estimation the number has been indicated.

1. All non-zero digits (1-9) are significant.
Examples: 44.55 — 4 significant digits; 0.6347 — 4 significant digits

2. Zeros that fall between two significant digits are significant.
Example: 12.0002 — 6 significant digits; 2004 — 4 significant digits

3. Leading zeros before the first non-zero digit are NOT significant. These zeros serve only as placeholders and do not represent measured data.
Examples: 0.00007 — 1 significant digit; 0.0725 — 3 significant digits

4. Zeros to the RIGHT of any non-zero digit are significant. The zeros represent actual measured data.
Examples: 0.0020000 — 5 significant digits; 7.0020 — 5 significant digits

B. Numbers without Decimals

In numbers without decimals, there is ambiguity concerning the estimated position. Zeros may be serving as placeholders or they may be representing measured data; there is no way to know. Scientific notation should always be used in these cases.

Examples: 52,000 may have 2, 3, 4, or 5 significant digits; there is no way to know
5.2×10^4 — 2 significant digits; 5.2000×10^4 — 5 significant digits

C. Exact Numbers

Exact numbers can be considered to have an infinite number of significant digits. Cardinal (or counting) numbers are exact numbers as are certain mathematical relationships or constants because of the way in which they are defined.

Examples: a dozen eggs is exactly 12 eggs; there are exactly 1,000 meters in a kilometer

Significant Digits in Computation

Since the numbers being used in computations may express varying degrees of uncertainty, it is important that the answer reflect the appropriate degree of uncertainty through the proper use of significant digits. Computation rules depend on the operation.

A. Rules for Addition and Subtraction

When quantities are added or subtracted, the number of *decimal places* in the resulting sum or difference is equal to the number of *decimal places* in the quantity with the smallest number of decimal places.

Example: $1.1111 + 378.34 = 379.45$

B. Rules for Multiplication and Division

When quantities are multiplied or divided, the number of significant digits in the resulting product or quotient is equal to the number of significant digits in the quantity with the smallest number of significant digits.

Example: $1.13 \cdot 10.1578 = 11.5$

C. Multiple Operations

In multiple operations, compute the number of significant digits to retain in the same order as the operations: exponents, then multiplication and division, and lastly addition and subtraction. Remember, however, to keep insignificant digits during intermediate calculations, rounding to the correct number of significant digits only in the final answer. (See Rounding Rules below.)

D. Rounding Rules

Use all available digits, significant or not, during intermediate calculations. Round to the nearest significant digit only when reporting your final answer.

1. Use the computation rules for significant digits to determine what the last reported digit should be.

2. Look at the digit to the right of the last digit to be reported.

3. If the digit to the right of the last digit to be reported is less than 5, drop it and all digits to the right of it.

4. If the digit to the right of the last digit to be reported is greater than 5, increase the last digit to be reported by one.

5. If the digit to the right of the last digit to be reported is equal to 5, drop it and all digits to the right of it, AND

   a. If the last digit to be reported is even, leave it as it is;

   b. If the last digit to be reported is odd, increase it by one (round up to the next even digit)

Examples: If rounding to 3 significant digits, 16.4376 becomes 16.4; 11.4728 becomes 11.5; 9.3659 becomes 9.36; and 3.875 becomes 3.88.

Reference:

Some of the material for this handout is adapted from: *Significant Figures Tutorial* located at
http://ist-socrates.berkeley.edu/~chem1a/sigfigs/sigfigs.htm