# Book Review

## An Introduction to Probability and Inductive Logic By Ian Hacking

## c. 2001 Cambridge University Press

Reviewed By Reiff Lafleur

For a while now, I have been trying to understand the nature of the controversy between the classical approach to statistical inference and the Bayesian approach. The classical approach uses confidence intervals to speculate about where a parameter lies. (e.g., a 95% confidence interval for a parameter is an interval for which one can be 95% confident that the parameter is contained in the interval. Alternatively, if many such confidence intervals are computed, 95% of them will contain the parameter.) Advocates of the classical method deny the meaning of statements about the probability that the parameter lies in any particular interval, since the parameter has a definite (though undetermined) value, and it is therefore either definitely inside or definitely outside of any given interval. For proponents of the Bayesian approach, probabilities refer to degrees of belief. Proponents contend that one can propose a (prior) probability distribution which represents one's present knowledge about the parameter's location. After acquiring additional data, one can then use Bayes' Theorem to develop a new (posterior) probability distribution, which represents the probable locations of the parameter in light of the new data.

These ideas (and more) are discussed in the fascinating book under review. I bought this book because the author was recommended as one who could shed some light on these issues. I was not disappointed. This is a well-written introduction to the ideas of probability. The author includes many brief biographical sketches of the main contributors of the various concepts. Many illuminating examples throughout the book clarify the concepts presented. After a brief discussion (for comparison purposes) of deductive logic, the author describes induction (scientific, not mathematical) and "inference to the best explanation." Although the author intends to discuss at length only induction, the exposition puts the subject matter into context. The author has a nice explanation of independence and randomness. He also briefly discusses the nature of the modeling process. Next comes a description of the basic concepts of probability and expected value. The St. Petersburg Paradox is discussed and several possible explanations are introduced to make sense of it. The St. Petersburg Paradox involves calculating the expected value of a game in which a coin is tossed until the first head appears. If the first head occurs on the first toss, a payoff of two dollars is made. If the first head occurs on the second toss, a payoff of four dollars is made. In general, if the first head occurs on the $n^{th}$ toss, a payoff of $2^n$ dollars is made. The expected value of the payoff in this game is infinite. But, as the author points out, no one would actually pay very much to play this game. One of the features of the book that I find appealing is the author's discussion of many different perspectives without singling out any one of them as being most deserving of acceptance.

Next, the two concepts of frequency-type probabilities and belief-type probabilities are presented. The author takes an eclectic approach to probability, stressing benefits of each of the two different approaches. The argument is made that degrees of belief must satisfy the axioms of probability, since if they do not, they are open to a sure loss contract. A *sure loss contract* is a set of bets that the person holding these beliefs would be expected to be willing to make, based on his beliefs, which would result in his losing money regardless of the outcome of the experiment. A nice application of the Bayesian approach to the diagnosis of a patient's condition is presented. This completes the discussion of the Bayesian approach to inductive reasoning.

The author now turns to a more classical approach. First Bernoulli's Theorem is presented. The relationship between the binomial distribution and the normal distribution is presented. Confidence intervals and tests of hypothesis are discussed. This treatment is along standard lines.

The final three chapters address the philosophical problem of induction: "How do we know that the future will be like the past?" This problem was posed by David Hume in the middle 1700's. I do not find the authors treatment of this question to be satisfactory, but then again, if someone were to deny the validity of reasoning from the past to the future, I do not know how one could answer their skepticism. It seems that the best argument to be made is that such reasoning has proved beneficial in the past. But it seems that this argument would not be found to be convincing to such a skeptic.

Overall I found the book to be very entertaining and informative. Many interesting ideas are discussed and the book would be a very valuable addition to the library of anyone interested in probability. I imagine that most instructors would not find it suitable as a textbook, though. It is too elementary for a Mathematical Statistics course, and even though (perhaps because) the book contains some material not often considered in an elementary class, there are some standard topics that are omitted, such as regression, analysis of variance, and tests of hypothesis involving comparisons between population means.

Department of Mathematics
Troy University (Dothan Campus)
Dothan, AL
rlafleur@troy.edu