Problems

Problem 1. Find the volume of a tetrahedron with corners at $(0, 1, 2)$, $(3, 5, 7)$, $(4, 6, 9)$, and $(8, 10, 11)$.

Problem 2. Buildings A and B are separated by a 12 foot wide alley. One 15 foot ladder rests at the base of building A and leans against the wall of building B. Another 20 foot ladder rests at the base of building B and leans against the wall of building A. What is the height where the two ladders cross?

Problem 3. A point is chosen at random from the interior of a circle of radius R. Determine the expected distance from the chosen point to the center of the circle.

Problem 4. Determine the maximum possible value of $xy^2 + yz^2 + zx^2$ such that $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + y + z = 1$.

Problem 5. Let C_1, C_2, and C_3 denote three circles with distinct radii whose interiors are pairwise disjoint. Let P_{ij} denote the point of intersection of the two external tangents of C_i and C_j. Prove that P_{12}, P_{13}, and P_{23} are collinear.

Problem 6. Find three different right triangles with integer length sides such that each has an area of 840.

Problem 7. Given that $F(x) = \tan(x)$, prove that the k^{th} derivative $F^{(k)}(0) \geq 0$ for every $k \geq 0$.

Problem 8. Three pairwise externally tangent circles have radii 26, 52, and 78 respectively. Determine all possible values for

the radius of a fourth circle that is tangent to all three of these circles.

Problem 9. A pair of ordinary dice is repeatedly tossed. Player A wins if the sum of the two dice is 12. Player B wins if a sum of 7 is obtained on two consecutive tosses. What is the probability that player A will win before player B?

Problem 10. Let C_1 and C_2 denote circles whose diameters coincide with the legs of a right triangle, and let C_3 denote a circle whose diameter coincides with the hypotenuse. Determine the relationship between the area of the triangle and the areas of the crescents of C_1 and C_2 that lie outside C_3.

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