Problems

Problem 1.

Four cars travel along a road at constant speeds. Cars A and B travel north, and cars C and D travel south. Car A meets car C at 1:00. Car A meets car D at 1:20. Car B meets car C at 2:00. Car B meets car D at 2:10. If cars C and D travel at identical speeds, what time will car B overtake car A? Alternatively, if cars A and B travel at identical speeds, what time will car D overtake car C?

Problem 2.

Mr. Jones wishes to add one-foot square tiles to his porch floor. He selects a pattern that consists of a central rectangle of blue tiles surrounded by red tiles along the outer perimeter. Mrs. Smith also wishes to add one-foot square tiles to her porch floor. She selects a pattern that consists of a central rectangle of red tiles surrounded by blue tiles along the outer perimeter. Mr. Jones' porch floor is 2 feet wider and 6 feet shorter than Mrs. Smith's porch floor. Coincidentally, both porches require identical numbers of blue tiles, and also identical numbers of red tiles. Determine the dimensions of both porch floors.

Problem 3.

To multiply binomials

$$(ax+b)(cx+d) = (ac)\,x^2 + (ad+bc)\,x + bd,$$

it initially appears as though 4 scalar multiplications will be needed. However, show how to multiply these binomials using only 3 scalar multiplications, plus any number of scalar additions and scalar subtractions.

Problem 4.

To multiply 2×2 matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix},$$

it initially appears as though 8 scalar multiplications will be needed. However, show how to multiply these 2×2 matrices using only 7 scalar multiplications, plus any number of scalar additions and scalar subtractions.

Problem 5.

Compute the value of $G(4,6)$ for the function $G(x,y)$ that is defined recursively below. For extra credit, also find a closed formula for $G(x,y)$; that is, provide a non-recursive definition of $G(x,y)$.

$$G(x,y) = \begin{cases} 1 & x=0,\ y=0 \\ 2G(x,y-1) & x=0,\ y>0 \\ 3G(x-1,y) & x>0,\ y=0 \\ G(x,y-1)+G(x-1,y)-G(x-1,y-1) & x>0,\ y>0 \end{cases}$$

Solutions, comments, and discussions should be sent no later than April 1, 2002, to:

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