A Mathematical Model for Optimum Water Distribution In a Remote City with a Water Tower

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Abstract

A mathematical model will be developed for simulating water distribution in Alabama remote city systems that have water towers. The objective of the proposed model is to optimize water delivery in remote city systems. The inputs to the model include the water source data, distribution facility capacity, delivery site networks, and other constraints. The output will be an optimized network of delivery routes, piping systems, optimum pump/power requirements, efficient delivery rates and use requirements. This will also maintain maximum capacity, lowest power requirements, greatest reliability and shortest down time. We assume that there is a constant pressure at the distribution point where the water tower is situated.

The objective functions to be optimized can be any or all of the following: smoothness of operation, lowest power, optimum pumping capacity, reliability, lowest down time, and total cost. Virtually any other efficiency function can be included. In the objective function, practical constraints can also be used including the following: size of available pumps, pipe diameters and lengths, limitations on station capacity, and number of delivery sites (virtually any others).

Critical Need Statement

Efficient water delivery from a distribution control center to a large number of delivery sites is currently managed without proper consideration of the water flow hydrodynamics. Costs are high, reliability is compromised, and large numbers of personnel are required for smooth operation. Customer supply needs are increasing rapidly. A hydrodynamics-based mathematical model can optimize the delivery system by running parametric cases overnight and automatically designing a more efficient overall delivery process. The software developed herein has the potential for designing systems which solve each of these current problems.

Results and Benefits

The proposed model will be research-based, but we will develop a mathematical model for a practical application. The results are expected to be an optimization model of water delivery systems. The benefits include lower overall costs, increased reliability, and more efficient operation of facilities and satisfied customers.

Nature, Scope and Objectives

The objectives of this research are to develop, code, and apply an aerospace methodology to the flow of water in a city delivery system. A finite element optimization method used by engineers in the aerospace industry will be applied to modeling city water delivery networks. The following Tasks will be required:

- Task 1. Modify Governing Equations for Water Flow including Incompressible Navier-Stokes and Adjoint Equations.
- Task 2. Write Visual Basic code implementing optimization algorithm.
- Task 3. Adapt existing aerospace solvers to water flow optimization.
- Task 4. Compute a test case for a typical Alabama city system and compare to data.

Methods, Procedures and Facilities

The following section gives details of the hydrodynamics modeling to be used in this project. We give the basics of a finite element technique, the hydrodynamics equations, the optimization equations, and the computer software plan. The research is based on previous work done in References 1-5.

Finite Element Model

Figure 1 shows a schematic of a generic water distribution control center. The basic components are the water sources, the control center, the delivery network, and the delivery sites. The parameters in the problem include: number and size of water sources, storage capacity of control center, power and pumping capability, number of delivery sites and daily water requirements of each site.

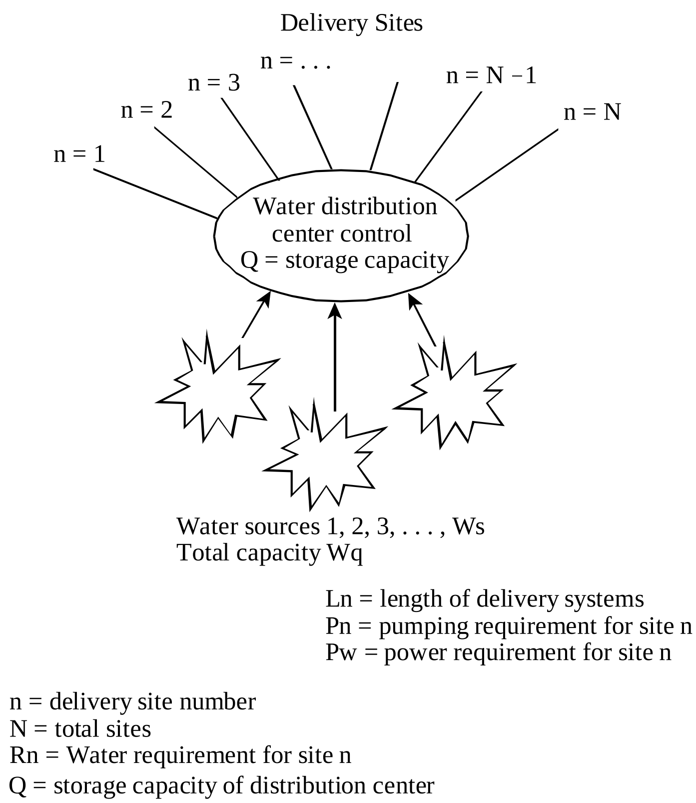

Figure 1

Figure 2 is for illustration of a typical finite element model. The actual model is complex and is computer generated. Each component of the generic system is "discretized" into a fixed, and finite, number of cells called elements. The parameters of the problem are approximated on the GRID of points. See References 3-4.

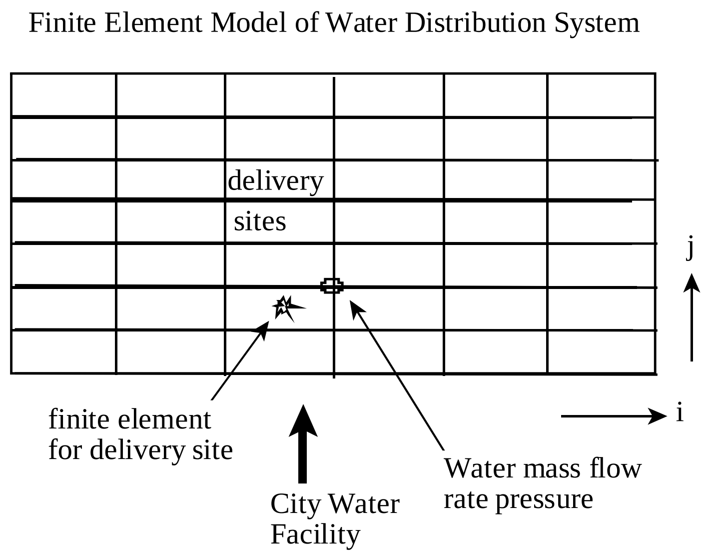

Figure 2. Finite Element Model of Water Distribution System

The relative size of the grid is important as the accuracy of the model increases and the number of elements increases. A typical model will contain a few thousand elements for this case. For some aerospace solutions, we have used 1 Million elements [1], and the fluid flow modeling via a stream function [5].

Hydrodynamics Equations

The Governing equation for the hydrodynamics of water flow is given by the incompressible Navier-Stokes equations. For this modeling effort, we use the time dependent, one space dimension set of equations. With the following definition of variables, the hydrodynamics equation is given below.

$M = \rho U =$ mass flow rate of water
$U =$ velocity flow
$\rho =$ density of water
$P =$ water pressure
$\mu =$ viscosity of water
$t =$ time
$x =$ distance

Navier Stokes Equation: $\frac{\partial M}{\partial t} + U\frac{\partial M}{\partial x} = -\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 M}{\partial x^2}$

Optimization Equations

The optimization algorithm is developed by defining an "objective function" with a set of "constraints". The parameters in the optimization problem are:

$$\begin{aligned} w_i &= \text{importance function} \\ f_i &= \text{efficiency objective function} \\ g_i &= \text{constraints} \\ \lambda_i &= \text{Lagrange multiplier} \end{aligned}$$

The objective function to be optimized is then given as follows:

$$F = \sum_{i=1}^{k} w_i f_i + \sum_{j=1}^{m} \lambda_i g_i$$

where k is the number of efficiency functions, and m is the number of constraints. The optimum is reached by using a mathematical minimization principle obtained as follows:

$$\frac{\partial F}{\partial f_i} = 0$$

The functions f_i are referred to as the *efficiency parameters* and can include such items as:

f_i $(i =)$	definition
1	a measure of smoothness of operation
2	lowest power usage
3	optimum pumping capacity
4	greatest reliability
5	total cost

Likewise, the constraint functions g_i can include:

g_i $(i =)$	definition
1	maximum allowed power
2	size of available pumps
3	manufacturablility of components
4	limitations on storage capacity
5	finite number of delivery sites

The weights w_i are called the "importance parameters." They can be assigned constant values between 0 and 1 according to their perceived importance. For example, each can be set to 0.20 and give equal importance to all variables. Setting one of them to zero gives it no importance, etc. The sum of parameters must be 1.0. The parameters λ_i are termed *Lagrange multipliers* and are used in classic mathematics to enforce constraints in the calculus of variations.

References

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