# Problems

(1) Let gcd denote greatest common divisor and lcm denote least common multiple. Prove that $\gcd(m,n) \times \operatorname{lcm}(m,n) = m \times n$ for all positive integers $m, n$.

(2) Recall that the sequence $\{1, 2, 3, \ldots, n\}$ has $n!$ permutations. Let $A_n$ denote the number of these permutations that have the property that no element appears in its original position. Determine $A_0, A_1, A_3, A_4, A_5$. Prove that $A_n = (n-1)(A_{n-1} + A_{n-2})$. Prove that $A_n = nA_{n-1} + (-1)^n$. Prove that $\lim_{n\to\infty} \frac{A_n}{n!} = \frac{1}{e}$. Find a non-recursive expression for $A_n$.

(3) A school purchases 20% of its computers from company X, 30% from company Y, and 50% from company Z. Experience shows that 5% of computers from company X are defective, 4% from company Y are defective, and 3% from company Z are defective. If the school discovers that a randomly chosen computer is defective, what are the probabilities that it was provided by company X, by company Y, and by company Z?

(4) Show that these three expressions are all logically equivalent:

(a) $(\exists x)(\forall y)(P(y) \leftrightarrow x = y)$.

(b) $(\exists x)P(x) \wedge (\forall x)(\forall y)((P(x) \wedge P(y)) \rightarrow x = y)$.

(c) $(\exists x)P(x) \wedge (\forall y)(P(y) \rightarrow x = y)$.

(5) Count how many permutations of the 26-letter alphabet do not contain any of these consecutive 6-letter substrings: $abcdef$, $fghijk$, $klmnop$, $pqrstu$, $uvwxyz$.

(6) The Fibonacci numbers are defined by the recurrence $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$.Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. Evaluate:

$$\sum_{k=0}^{\infty} \frac{F_k}{3^k} \quad \text{and} \quad \sum_{k=0}^{\infty} \frac{F_k}{k!}.$$

(7) Five consecutive houses are painted distinct colors and are inhabited by five people with distinct nationalities, distinct careers, distinct pets, and distinct favorite foods. Given the facts below, determine all the attributes of each house and its inhabitant.

(a) The American owns a cat.

(b) The Frenchman lives in the white house.

(c) The yellow house is on the immediate right side of the blue house.

(d) The Mexican eats hamburgers.

(e) The Chinese man lives in the first house on the left.

(f) The doctor breeds mice.

(g) The Chinese man's house is next to the red one.

(h) The Indian man is a mathematician.

(i) The alligator is in a house next to the one owned by the lawyer.

(j) The artist lives in the green house.

(k) The owner of the yellow house eats pizza.

(l) The biologist eats ice cream.

(m) The goldfish is in a house next to that of the artist.

(n) Salad is eaten in the middle house.

(o) The person who eats macaroni is not the owner of the parrot.

(8) Prove that $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} + \ldots = \frac{\pi^2}{6}$.

Solutions, comments, and discussions should be sent to:

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