Problems

(1) Determine a function $F(n)$ such that $F(1) = 10$, and $F(n) = 4F\left(\frac{n}{2}\right) + 3n^2$ for $n \geq 2$. If it helps, you may assume that n is a power of 2.

(2) Develop a formula for the maximum number of regions that can be formed by using N circles to partition the plane. Also, what implications does this have for Venn diagrams?

(3) Let M_n denote the $n \times n$ matrix such that each element $m_{ij} = 1$ when $|i - j| \leq 1$, and otherwise $m_{ij} = 0$. Compute the determinant of matrix M_n for all $n \geq 1$.

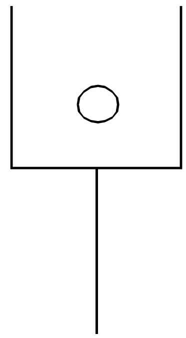
(4) Determine whether each of these three statements is True or False.

  (a) The number of True statements in this list is even.

  (b) The number of False statements in this list is odd.

  (c) If statement 4a is True then statement 4b is True.

(5) On the following page are four toothpicks and an olive. (The toothpicks are arranged in the shape of a cocktail glass.) Move two toothpicks so that the olive will appear to be outside the glass, but so that the glass retains the same shape.



(6) Consider A, the $n \times n$ matrix with 2's on the main diagonal, 1's on the neighboring diagonals, and 0's everywhere else. (The matrix is shown below.) Prove that for all natural numbers n, the eigenvalues of A are given by $\lambda_i = 2 + 2\cos\left(\frac{i\pi}{n+1}\right)$ for $i = 1, 2, \ldots, n$.

$$\begin{bmatrix} 2 & 1 & 0 & & \cdots & & 0 \\ 1 & 2 & 1 & 0 & \cdots & & 0 \\ 0 & 1 & 2 & 1 & \ddots & & \vdots \\ & 0 & 1 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 2 & 1 & 0 \\ & & & \ddots & 1 & 2 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 2 \end{bmatrix}$$

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