Mathematical "Narrow-Casting" - For Business Calculus

By Pat Rossi

Introduction

This paper chronicles the evolution of my teaching of Business Calculus, arrived at pragmatically. Consider the following e-mail conversation from 1999.

Hello again, Stephen:

I've enclosed the Articulation and General Studies Committee's description of the topics that all Business Calculus courses, approved by the committee, should contain. What do you think of it?

Pat

MA 120 - - Calculus and Its Applications (3 semester hours)

This course is intended to give a broad overview of calculus and is taken primarily by students majoring in Commerce and Business Administration. It includes differentiation and integration of algebraic, exponential, and logarithmic functions and its applications to business and economics. The course should include functions of several variables, partial derivatives (including applications), Lagrange Multipliers, L'Hopital's Rule, and Multiple Integration (including applications).

Prerequisite: MA 112 Precalculus Algebra

(1/15/99)

Pat,

Why don't we include "Discusses origin, structure, future, and meaning of the universe and all it includes?"

Stephen

Below, we list the topics to be covered in the course.

(1) Limits
   (a) Limits
   (b) Continuity
   (c) One-Sided Limits
   (d) Limits at Infinity
   (e) Infinite Limits
(2) The Derivative
   (a) Derivative by the Definition (using limits)
   (b) Rules of Derivatives
   (c) Product and Quotient Rules
   (d) The Chain Rule & the Derivative of a Function to a Power
   (e) Rates of Change
   (f) Marginal Analysis
   (g) Higher Order Derivatives
   (h) Implicit Derivatives
   (i) Related Rates
(3) Applications of Derivatives
   (a) Increasing/Decreasing Functions & Critical Numbers
   (b) Relative Extremes & Curve Sketching
   (c) Concavity/2nd Derivative Test
   (d) Absolute Extremes
   (e) Applied Max/Min Problems
   (f) Elasticity of Demand
(4) Exponential & Logarithmic Functions
   (a) Introduction to Logarithmic Functions
   (b) Introduction to Exponential Functions
   (c) Differentiation of Logarithmic Functions
   (d) Differentiation of Exponential Functions
(5) Integration
   (a) Antiderivatives
   (b) Differentials
   (c) Integration by Substitution
   (d) The Definite Integral and Area Under a Curve
   (e) The Fundamental Theorem of Calculus
   (f) Applications - Consumer Surplus/Producer Surplus

(6) Miscellaneous
   (a) L'Hopital's Rule
(7) Multivariable Calculus
   (a) Functions of Two or Three Variables
   (b) Partial Derivatives
   (c) Local Extremes
   (d) Lagrange Multipliers
   (e) Double and Triple Integrals
   (f) Applications of Multiple Integrals

That's 37 topics in one semester!

Given that the "typical class" (i.e., one that meets for one hour, three times per week) meets 41 times per semester, and allowing for four tests per semester, that leaves 37 hours in which to cover 37 topics. Said differently, the course outline allows for one class hour for each topic on the outline.

So What's the Problem?

For the sake of comparison, note that in the "regular Calculus sequence," (which meets for one hour, *four* times per week) the topics of Limits, The Derivative, Applications of Derivatives, Exponential & Logarithmic Functions, Integration, and Multivariable Calculus require close to TWO semesters to cover.

True, the "regular Calculus sequence" includes certain topics not covered in Business Calculus, such as the derivatives and integrals of Trigonometric functions; arc-length; solids of revolution; work problems; etc. But Business Calculus, of necessity, covers certain topics not covered in "regular Calculus," such as: Marginal Analysis, Elasticity of Demand, Consumer Surplus/Producer Surplus, and Applications of Multiple Integrals. Add to this the fact that by any honest assessment, students who take "regular Calculus" have, on the whole, mathematical skills that are vastly superior to those of Business Calculus students.

Hence, our dilemma: How can Business Calculus students learn, in one semester, material that students with superior mathematical ability require TWO semesters to learn?

Our Challenge

Our dilemma presents those of us who teach Business Calculus with a formidable challenge: Try to be as faithful as possible to the "syllabus" established by the Alabama State Articulation Committee, and, at the same time, teach the topics in such a way that our

Business Students understand and retain, as fully as possible, the information, concepts, and techniques.

Our Solution(s)

To meet our challenge, I propose the following solution — one that I have implemented with great success.

(1) Eliminate the teaching of limits entirely.
(2) Omit all other topics not relevant to BUSINESS Calculus

To justify this approach (which some might consider to be mathematical heresy), consider some problems that are typical of the problems that I might put on my Business Calculus final exams, IF I adhered strictly to the State Articulation Committee's syllabus.

(1) Compute: $\lim_{x\to 0} \frac{2-\sqrt{4+x}}{x} =$

(2) Compute: $\lim_{x\to 2} \frac{x^3-2x^2+2}{x^3+3} =$

(3) Find asymptotes and graph
$f(x) = \frac{2x+5}{3-x}$

(4) Compute: $\lim_{x\to 2} \frac{x^2-4}{x^2+2x-8} =$

(5) $f(x) = x^2 + 2x$; compute $f'(x)$ using the definition of. derivative.

The sad facts of the matter are:

(1) Most Business Calculus students never grasp the concept of limits.
(2) Business Calculus students' inability to grasp the concept of limits seems to pose no barrier to their being able to compute and work with derivatives. Specifically:
    (a) Most students who perform well in the course "bomb" the section on limits on the final exam. (This includes the "A" students).
    (b) Everything that students learn in Business Calculus, they learn without the concept of limits.

From this we can conclude that the section on Limits serves no purpose. It just takes up time that could be spent teaching the essential topics more thoroughly. As a bonus, it might be worth noting that if we don't cover Limits, then we won't be able to cover L'Hopital's Rule later on in the course. But in light of the fact that L'Hopital's Rule is used, almost exclusively, to determine convergence/divergence of sequences and series — something that wasn't in State Articulation Committee's Business Calculus syllabus in the first place — we can justify deleting L'Hopital's Rule from the course outline.

Execution

The question which naturally arises at this point is: "How do we implement the teaching of this 'Calculus without limits'?"

(1) Begin the course with a brief review of slope and lines (a **review**, not a re-teaching).
(2) Define the concept of *tangent,* using pictures

Example 1. *Def - Given the graph of a function,* $f(x)$, *and a point,* $(x_1, f(x_1))$, *on the graph, the* line tangent *to the graph at the point* $(x_1, f(x_1))$ *is the line containing the point* $(x_1, f(x_1))$ *and having the same slope as the graph of* $f(x)$ *at that point.*

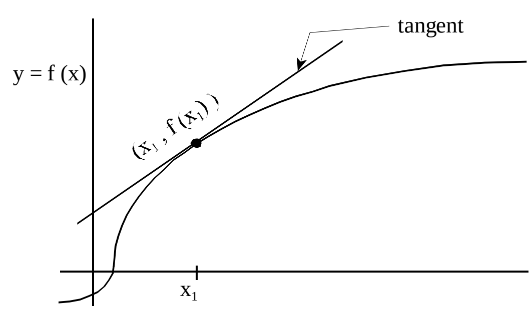


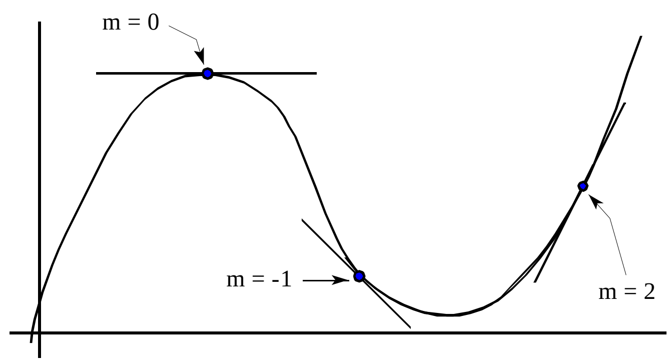
Fig. 1

At this point, it is appropriate to explain the *main goal* of the course, and present it as such, writing it on the board:

Main Goal: Given any function, $f(x)$, and any point $(x_1, f(x_1))$ on the graph of $f(x)$, compute the slope of the line tangent to the graph of $f(x)$ at the point $(x_1, f(x_1))$.

At this point in the lecture, it's only natural for students to wonder: "Why do we want to do this — Why is this important?" Assure students that, at this point in the course, there's no reason why they should understand why this would be an important thing to know. For the time being, they should just accept the claim that this *is* important.

Give some graphic examples:



3. Introduce the "$\frac{d}{dx}$ notation" and then introduce the rules for computing derivatives.

Further Refinements

(1) Chain Rule
   (a) Limit the initial treatment to that of computing the derivative of a function to a power.

   i.e. $\dfrac{d}{dx}\left[(g(x))^n\right] = n\,(g(x))^{n-1}\,g'(x)$

   One justification for this is that if the general form of the Chain Rule is taught at this point, the students will have forgotten it by the time they have a chance to apply it to exponential and logarithmic functions.

(2) Higher Order Derivatives
   (a) Don't do this.
   Since Business Calculus is not a "theory course," and since derivatives of order higher than the 2nd Derivative find their application almost exclusively in Taylor Series, which is not covered in Business Calculus, we don't cover higher order derivatives.

(b) What about the 2nd Derivative?
We only need this for concavity and the 2nd Derivative Test (discussed later).

(3) Implicit Derivatives

(a) Again, Business Calculus is not a "theory course," so inclusion of this topic in Business Calculus is, at best, questionable. Furthermore, consider that the main application of Implicit Differentiation after its introduction is that of deriving formulas for the derivatives of certain transcendental functions (e.g., $e^x, \sin^{-1}(x)$, etc.). Consequently, we should omit Implicit Differentiation.

(4) Related Rates

(a) Again, this is a "theoretical topic" - Business Calculus is NOT a "theory course." We omit Related Rates.

(5) Concavity/2nd Derivative Test

(a) I've found the inclusion of this topic in a Business Calculus course to be a mistake.

Pros

(i) Knowledge of the 2nd Derivative is useful in curve sketching.

(ii) Knowledge of concavity sometimes comes in handy when doing applied extrema problems (where the domain is NOT a closed interval of finite length).

Cons

(i) While the 1^{st} Derivative Test ALWAYS works, the 2^{nd} Derivative Test can fail. Consequently, the 2^{nd} Derivative Test enables students to do nothing that they can't already do with greater success using the 1^{st} Derivative Test.

(ii) Students in "regular Calculus" are sometimes prone to confusing the 2^{nd} Derivative Test with the 1^{st} Derivative Test. (e.g., getting critical numbers from $f''(x)$ instead of from $f'(x)$).

(iii) The association:

$$\begin{array}{lcl} f''(c) \text{ positive} & \Rightarrow & \text{relative max} \\ f''(c) \text{ negative} & \Rightarrow & \text{relative min} \end{array}$$

although incorrect, is much stronger and natural than:

$$\begin{array}{llllll} f''(c) > 0 & \Rightarrow & \text{concave up} & \Rightarrow & \text{rel. min} \\ f''(c) < 0 & \Rightarrow & \text{concave down} & \Rightarrow & \text{rel. max} \end{array}$$

Regardless of the warnings given by the instructor and the steps taken by the instructor to prevent this type of thing from happening, Business Calculus Students are much MORE prone to making these mistakes than are students in "regular Calculus." Worse still, students in Business Calculus have a tendency to propagate these errors back into other max/min tests, such as:

(A) 1st Derivative Test. (e.g., get the critical number from $f'(x)$, and then draw a sign graph of $f''(x)$ to determine where $f(x)$ is increasing/decreasing.
(B) Plugging critical numbers into $f''(x)$ to get the y-coordinates of relative maxes and mins,
(C) etc.

The bottom line?
The 2nd Derivative Test accomplishes nothing, other than to confuse the students — even the *better* Business Calculus students are prone to making these mistakes.

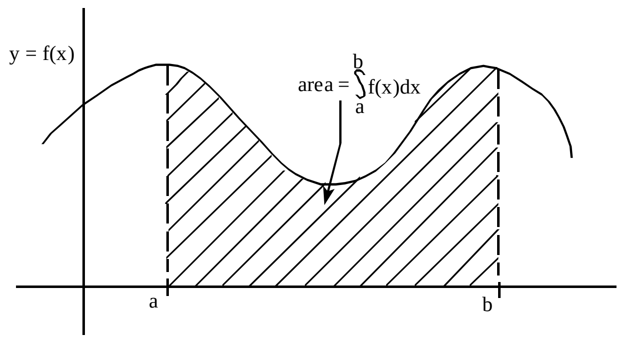
(6) Derivatives of Exponential and Logarithmic Functions
  (a) Cover these without any introduction to the Chain Rule.

$$\text{e.g., } \frac{d}{dx}[\ln(g(x))] = \frac{1}{g(x)}g'(x)$$

Justification: e^x and $\ln(x)$ are the only functions that Business Calculus students encounter, that require the application of the "general" Chain Rule. Instead of spending all of the time to teach the concept of the Chain Rule, why not just teach the students how to compute the derivatives of these functions "by rote."

(7) The Definite Integral/Area Under a Curve/Fundamental Theorem of Calculus

(a) Skip any mention of "Riemann Sum" - this is NOT a "theoretical course." Tell the students: "Given a 'continuous' function, $f(x)$, the area bounded by the graph of $f(x)$ and the x-axis, between $x = a$ and $x = b$, is given by the **Definite Integral of** $f(x)$ **from** a **to** b, denoted $\int_a^b f(x)\,dx$.



Then state the Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

and give examples.

The Refined List of Topics

(1) Review of Slope and Lines
(2) The Derivative
  (a) Rules of Derivatives
  (b) Product and Quotient Rules
  (c) Derivative of a Function to a Power
  (d) Rates of Change
  (e) Marginal Analysis
(3) Applications of Derivatives
  (a) Increasing/Decreasing Functions & Critical Numbers
  (b) Relative Extremes & Curve Sketching
  (c) Absolute Extremes
  (d) Applied Max/Min Problems
  (e) Elasticity of Demand
(4) Exponential & Logarithmic Functions
  (a) Introduction to Logarithmic Functions
  (b) Introduction to Exponential Functions
  (c) Differentiation of Logarithmic Functions

    (d) Differentiation of Exponential Functions
(5) Integration
    (a) Antiderivatives
    (b) Differentials
    (c) Integration by Substitution
    (d) The Definite Integral/Area Under a Curve/Fundamental Theorem of Calculus
    (e) Applications - Consumer Surplus/Producer Surplus
(6) Multivariable Calculus (Time Permitting)
    (a) Functions of Two or Three Variables
    (b) Partial Derivatives
    (c) Local Extremes
    (d) Lagrange Multipliers
    (e) Double and Triple Integrals
    (f) Applications of Multiple Integrals

Not counting Multivariable Calculus, that's a bare minimum 20 topics in one semester, which allows for almost two class hours per topic. True, some topics may only take an hour (or less) to cover. But others (e.g., Applied Max/Min Problems) may take considerably longer! For this reason, we leave the topics in Multivariable Calculus as "Time Permitting."

FAQ's

(1) Some topics (e.g. Extreme Value Theorem, and Definite Integrals) require the notion of continuity. How do we handle these topics if we omit limits and continuity from the course?

    Answer: We define a *continuous function* on the interval $[a, b]$ as one that has no x-values between $x = a$ and $x = b$ that cause division by zero. Naive? Absolutely. But keep in mind that regardless of the approach that we use to teach Business Calculus, our students are not apt to gain a more sophisticated understanding of the topic.

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