An Algebraic Analysis Of The Value Of Hay Silage: An Example Of Multiple Approaches To A Single Problem

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Teachers are constantly looking for situations in which algebra can be connected to real world issues. We present one such setting.

A farmer in our area stored 160 tons of hay silage (hay which is cut and stored without being dried) in his silo. A fire subsequently destroyed his silage, and the insurance company was called to adjust the loss.

A problem arose because there is not an established market price for hay silage; it is commonly used on the farm and is usually not sold. To find the value of the silage, we need to compare it to dried hay (hay cut and dried before being stored). We know:

- One ton (2000 pounds) of dried hay has 16% moisture (water content) and costs \$120.
- One ton of hay silage has 45% moisture.

What should be the price of silage? Encourage your students to find different methods for solving the problem. Three possible methods are given here.

Method 1: Comparing by Dry Matter

Two thousand pounds of dried hay contains 84% of 2000 or 1680 pounds of dry matter. The dry matter price is then $\$\frac{120}{1680} = \$0.0714$ per pound of dry matter.

However, 2000 pounds of silage contains 55% of 2000 or 1100 pounds of dry matter. Consequently, 160 tons of silage contains $(160)\,(1100) = 176,000$ pounds of dry matter. The value of this dry matter contained in this silage is $(176,000)(\$0.0714) = \$12,566$.

Method 2: Converting Silage to "Dried Hay"

Some farmers may argue that Method 1 is theoretically true but is unrealistic since it is impossible to remove all the moisture from hay or hay silage. The dry matter used in Method 1 is an artificial construct.

To approach the problem in a less artificial manner, we will determine how many pounds of normal dried hay (16% moisture) could be attained by removing the excess water from 160 tons of silage (45% moisture). 160 tons of silage has $(160)\,(2000)(0.55) = 176,000$ pounds of dry matter and $320,000 - 176,000 = 144,000$ pounds of water. Since normal dried hay has 84% dry matter and 16% moisture, $\frac{\text{dry matter}}{\text{water}} = \frac{84}{16} = \frac{21}{4} = 5.25$. Thus, the allowable pounds of water (x) to accompany the 176,000 pounds of dry matter can be found as follows:

$$\begin{aligned} \tfrac{176{,}000}{x} &= 5.25 \\ 5.25x &= 176,000 \\ x &= 33,524. \end{aligned}$$

The number of *pounds* of normal dried hay contained in 160 tons of hay silage is thus 176,000 (dry matter) + 33,524 (water) = 209,524. This is equal to 104.762 tons. Consequently, its value is $(104.762)(\$120) = \$12,571$.

Method 3: Comparing Silage to Dried Hay Directly

For normal dried hay,

$$\frac{\text{total weight}}{\text{water weight}} = \frac{100}{16} = \frac{25}{4} = 6.25$$

Let x be the number of pounds of water to be removed from a ton of silage to produce normal dried hay. Since a ton of silage contains 900 pounds of water,

$$\begin{aligned} \tfrac{2000-x}{900-x} &= 6.25 \\ 2000 - x &= (6.25)\,(900 - x) \\ 2000 - x &= 5625 - 6.25x \\ 5.25x &= 3625 \\ x &= 690.5 \end{aligned}$$

Thus, one ton of silage reduces to $2000-690.5 = 1309.5$ pounds of dried hay, or $\frac{1309.5}{2000} = 65.5\%$ of a ton of dried hay. Consequently, one ton of silage hay should be worth 65.5% of the value of a ton of hay, or $(0.655)(120) = \$78.60$. The value of 160 tons of silage would thus be $(160)\,(78.60) = \$12,576$.

The difference in the value found by these three methods is caused by the number of digits used in rounding.

Some farmers may feel that Method 3 is the best approach since it avoids the artificial construct of dry matter altogether.

Challenges for the Reader and His/Her Students

(1) Find other approaches to solving the problem presented in this article.
(2) Find other situations in which algebra can be exploited.

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