

# Problems

- Problem 1.** How many regions can be created in a plane using  $N$  lines? How many regions can be created in a plane using  $N$  circles? How many regions can be created in 3-dimensional space using  $N$  planes? How many regions can be created in 3-dimensional space using  $N$  spheres?
- Problem 2.** Determine all values of  $N$  such that the sum  $1^2 + 2^2 + \dots + N^2$  is a perfect square.
- Problem 3.** Suppose point  $P$  lies inside equilateral triangle  $ABC$  and let the lengths  $PA = x$ ,  $PB = y$ , and  $PC = z$ . Find the lengths of the sides of triangle  $ABC$  expressed in terms of  $x$ ,  $y$ , and  $z$ .
- Problem 4.** Suppose point  $P$  lies inside square  $ABCD$  such that angles  $\angle PAB$  and  $\angle PBA$  each measure 15 degrees. Find the measure of angle  $\angle CPD$ .
- Problem 5.** Determine the number of distinct paths from point  $(0, 0)$  to point  $(N, N+k)$ , such that each path consists of moves to the east or north along lattice points  $(x, y)$  where  $x \leq y \leq x+k$ . [Hint: first consider particular small values of  $k$ .]
- Problem 6.** If points  $A, B, C$ , and  $D$  are chosen at random along a given circle, determine the probability that segment  $AB$  intersects segment  $CD$ . If points  $A, B, C, D, E$ , and  $F$  are chosen at random along a given circle, determine the probability that triangle  $ABC$  intersects triangle  $DEF$ .

**Problem 7.** How many total gifts are given in the song “The 12 Days of Christmas”? If additional verses are added in the same pattern, how many total gifts would be given in the song “The  $N$  Days of Christmas”?

Solutions, comments, and discussions should be sent to:

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