Geometry Examination
2003 ACTM — AACTM Alabama Statewide Mathematics Contest

Construction of this test directed by Scott H. Brown, AUM; diagrams and typesetting by Steve White, JSU.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

INSTRUCTIONS. We use the following geometric notation: If A and B are points, then $\overline{AB}$ is the segment between A and B, $\overleftrightarrow{AB}$ is the line containing A and B, $\overrightarrow{AB}$ is the ray from A through B, and AB is the distance between A and B. If A is an angle, then $m\angle A$ is the measure of angle A in degrees. If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B, $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees. Diagrams are *not* necessarily to scale.

For each problem a box has been drawn around the correct answer. Below each possible answer is the percentage of contestants who chose that answer.

(1) Each edge of $\triangle BCD$ has length 2, D lies on $\overline{AC}$, and $m\angle ABC = 90°$. What is the length of $\overline{AB}$?

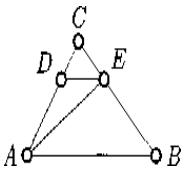
(A)	2	**(B)**	4	**(C)**	$3\sqrt{2}$	**(D)**	$\sqrt{3}$
	5%		9%		4%		3%
(E)	$\boxed{2\sqrt{3}}$	Omit					
	65%	14%					

(2) If $CO = 2$, $AB = 8$, and $OA = OB$, find CD.

(A) $\sqrt{38}$ 1% **(B)** 8 35% **(C)** [10] 40% **(D)** $2\sqrt{5}$ 2%

(E) 32 1% Omit 21%

(3) Given that $\overline{AB} \| \overline{DE}$ and $DE : AB = 1 : 3$, if the area of triangle $\triangle CDE$ is 20, then the area of triangle $\triangle DBA$ is:



(A) 20 8% **(B)** [40] 25% **(C)** 80 10% **(D)** 100 2%

(E) 120 3% Omit 53%

(4) What is the length of AB, given that $CD = 24$ and $EF = 18$?

(A) $\frac{24}{7}$ 4% **(B)** $\frac{36}{7}$ 4% **(C)** $\frac{52}{7}$ 4% **(D)** $\boxed{\frac{72}{7}}$ 49%

(E) $\frac{108}{7}$ 0% Omit 38%

(5) What is the volume of a sphere of radius 6?

(A) 48π 5% **(B)** 72π 9% **(C)** 144π 11% **(D)** $\boxed{288\pi}$ 44%

(E) 864π 3% Omit 28%

(6) In the right triangle $\triangle ABC$, the altitude from vertex C divides the hypotenuse into two segments, one of length 2 and the other of length 16. Find the perimeter of triangle $\triangle ABC$.

(A) $24 + 4\sqrt{14}$ 2% **(B)** $18 + 2\sqrt{7} + 12\sqrt{2}$ 10% **(C)** 36 3%

(D) $\boxed{24 + 12\sqrt{2}}$ 42% **(E)** $16\sqrt{14}$ 2% Omit 41%

(7) What is the maximum number of points of intersection of two different lines and three different circles in the same plane?

(A) 9 8% **(B)** 13 16% **(C)** $\boxed{19}$ 28% **(D)** 22 3%

(E) 25 3% Omit 42%

(8) A 5 inch by 8 inch rectangle is enlarged to a similar rectangle whose smallest side measures 9 inches. What is the length in inches of the diagonal of the enlarged rectangle?

(A) $\frac{5\sqrt{89}}{9}$ 4% **(B)** $\frac{9\sqrt{33}}{5}$ 2% **(C)** $\frac{5\sqrt{69}}{9}$ 5% **(D)** $\boxed{\frac{9\sqrt{89}}{5}}$ 37%

(E) $\frac{9\sqrt{69}}{5}$ 6% Omit 46%

(9) A bicycle has a 100 cm diameter wheel. If you ride on and around a circle with a 10 km diameter 12 times, how many revolutions does the wheel make?

(A) 12×10^2 16% **(B)** 12×10^3 13% **(C)** $\boxed{12 \times 10^4}$ 25%

(D) 12×10^5 10% **(E)** 12×10^6 6% Omit 30%

(10) A circle of radius 10 has its radius reduced by 4. By what percentage has its area been decreased?

(A) 36% 11% **(B)** 40% 28% **(C)** $\boxed{64\%}$ 50% **(D)** 70% 2%

(E) 80% 1% Omit 8%

(11) In triangle $\triangle ABC$, D bisects side $\overline{BC}$, G bisects side $\overline{AB}$, and the points E and F trisect side $\overline{AC}$. What is the area of the shaded polygon, if the area of $\triangle ABC$ is 108?

(A) 18 2% **(B)** 36 5% **(C)** 64 9% **(D)** $\boxed{72}$ 19%

(E) 90 5% Omit 60%

(12) What is the area of the triangle whose vertices are $(0, 0)$, $(14, 6)$, and $(12, 9)$?

(A) 9 1% **(B)** 18 3% **(C)** $\boxed{27}$ 47%

(D) 54 8% **(E)** 84 2% Omit 38%

(13) In a triangle $\triangle ABC$, $AC = 36$, $B = 48$, and the medians $\overline{BD}$ and $\overline{AE}$ to sides $\overline{AC}$ and $\overline{BC}$, respectively, are perpendicular. Find AB.

(A)	$\boxed{12\sqrt{5}}$	**(B)**	$15\sqrt{2}$	**(C)**	$6\sqrt{5}$	**(D)**	$5\sqrt{6}$
	10%		6%		5%		5%
(E)	$5\sqrt{12}$	Omit					
	2%	72%					

(14) What is the angle between the hour hand and minute hand of a clock face at 2 : 25?

(A)	$25\,^\circ$	**(B)**	$60\,^\circ$	**(C)**	$72\frac{1}{2}\,^\circ$	**(D)**	$\boxed{77\frac{1}{2}\,^\circ}$
	2%		4%		11%		46%
(E)	$80\,^\circ$	Omit					
	19%	18%					

(15) To construct a circle that circumscribes a triangle one finds its center by locating the intersection of which two lines?

(A)	Two Medians	18%
(B)	$\boxed{\text{Perpendicular bisectors of two sides}}$	29%
(C)	Bisectors of two angles	14%
(D)	Two altitudes	6%
(E)	None of the above	6%
Omit	Omit	26%

(16) An equilateral triangle is inscribed in a circle. Each side of the triangle has length x. What is the area of the circle?

(A)	$\frac{\pi x^2}{2}$	**(B)**	$\frac{3\pi x^2}{4}$	**(C)**	$\frac{\pi x^2}{4}$	**(D)**	$\boxed{\frac{\pi x^2}{3}}$
	7%		8%		6%		30%
(E)	πx^2	Omit					
	8%	41%					

(17) Given $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$, $AB = 9$, and $CD = 12$, if the area of triangle $\triangle CPD$ is 64, then the area of triangle $\triangle APB$ is?

(A) 9 1% **(B)** 12 2% **(C)** 18 6% **(D)** 24 13%

(E) 36 35% Omit 43%

(18) Three circles are mutually tangent externally. Their centers form a triangle whose sides are of lengths 8, 9, and 13. Find the total area of the three circles.

(A) 64π 2% **(B)** 89π 37% **(C)** 108π 7% **(D)** 229π 2%

(E) 314π 4% Omit 50%

(19) The length of a certain rectangle is quadrupled and the width is tripled. What is the ratio of the area of the new enlarged rectangle to the original rectangle?

(A) 3 : 1 5% **(B)** 7 : 2 4% **(C)** 4 : 1 6% **(D)** 5 : 2 2%

(E) 12 : 1 59% Omit 23%

(20) An equilateral triangle and a regular hexagon have equal perimeters. What is the area of the triangle, if the area of the hexagon is 120?

(A) 40 8% **(B)** 60 11% **(C)** 80 17% **(D)** 90 1%

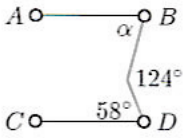
(E) 120 9% Omit 54%

(21) The larger angles of a rhombus are twice the smaller angle of the rhombus. If the shorter diagonal is 20, find the perimeter of the rhombus.

(A) 40 4% **(B)** $40\sqrt{3}$ 6% **(C)** 60 5%

(D) 80 40% **(E)** $80\sqrt{3}$ 2% Omit 42%

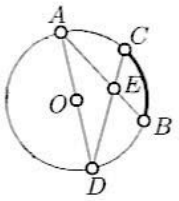
(22) Given that $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$, find the number of degrees in angle $\angle\alpha$.



(A) 12° 1% **(B)** 56° 5% **(C)** 58° 15% **(D)** 66° 58%

(E) 72° 3% Omit 17%

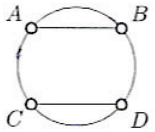
(23) Given that angle $m\angle A = 30°$, $m\angle D = 28°$, and $\overline{AD}$ is a diameter of the circle, find the number of degrees in arc $\overset{\frown}{BC}$.



(A) 28° 2% **(B)** 30° 1% **(C)** 58° 14% **(D)** 64° 50%

(E) 128° 8% Omit 25%

(24) Given that $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$, which of the following is true?



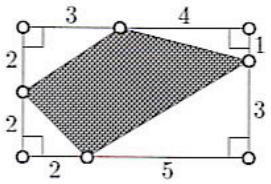
I. $m\overset{\frown}{AC} = m\overset{\frown}{BD}$ II. $m\overset{\frown}{AB} = m\overset{\frown}{CD}$
III. $m\overset{\frown}{AB} = m\overset{\frown}{BD}$ IV. $m\overset{\frown}{AB} = m\overset{\frown}{AC}$

(A)	I only	44%
(B)	II only	12%
(C)	I and IV only	2%
(D)	None of them	13%
(E)	All of them	16%
Omit	Omit	14%

(25) What is the number of different squares which can be inscribed in a given equilateral triangle?

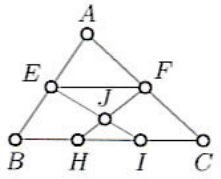
(A)	0 10%	**(B)**	1 26%	**(C)**	2 6%	**(D)**	3 19%
(E)	4 8%		Omit 30%				

(26) What is the area of the shaded portion of the figure?



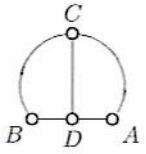
(A)	$13\frac{1}{2}$ 41%	**(B)**	$14\frac{1}{2}$ 10%	**(C)**	$19\frac{1}{2}$ 3%		
(D)	20 2%	**(E)**	39 1%		Omit 44%		

(27) In triangle $\triangle ABC$, E and F are midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively; and H and I trisect the side $\overline{BC}$. If the area of triangle $\triangle ABC$ is 120, what is the area of triangle $\triangle EFJ$?



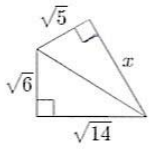
(A) 9 2%
(B) 18 6%
(C) 24 8%
(D) 30 11%
(E) 40 6%
Omit 66%

(28) In the figure, $A\overset{\frown}{C}B$ is an arc of a circle, and $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$. If $CD = 18$ and $AB = 12$, find the area of the entire circle.



(A) 36π 5%
(B) 81π 6%
(C) 100π 32%
(D) 144π 9%
(E) 324π 3%
Omit 45%

(29) What is the value of x?

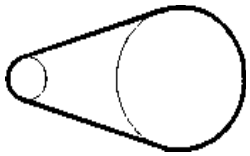


(A) $\sqrt{15}$ 63%
(B) 16 1%
(C) $16\frac{4}{5}$ 2%
(D) $\sqrt{12}$ 7%
(E) $2\sqrt{5}$ 4%
Omit 23%

(30) Find the perimeter in inches of a regular polygon whose area is $24\sqrt{3}$ inches, and whose apothem, the perpendicular distance from the center of the regular polygon to one of its sides, is $2\sqrt{3}$ inches.

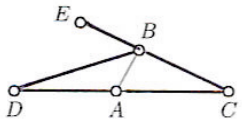
(A) 6 4%
(B) $12\sqrt{3}$ 7%
(C) 18 3%
(D) 24 32%
(E) $24\sqrt{3}$ 2%
Omit 52%

(31) The radii of two pulley wheels are 5 inches and 1 inch. The distance between their centers is 8 inches. Find the length of a pulley belt which passes around the two pulley wheels.



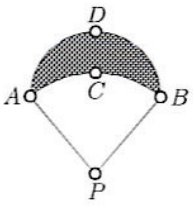
(A) $\frac{20\pi}{3}$ 2%
(B) $\frac{22\pi}{3} + 8\sqrt{3}$ 13%
(C) $\frac{28\pi}{3}$ 5%
(D) $\frac{20\pi}{3} + 4\sqrt{3}$ 4%
(E) $\frac{40\pi}{3}$ 2%
Omit 74%

(32) In the given figure, $\overline{DB}$ bisects the exterior angle $\angle EBA$ of triangle $\triangle ABC$. If $AB = 6$, $BC = 10$, and $AC = 12$, find DA.



(A) 12 25%
(B) 14 3%
(C) 16 3%
(D) 18 4%
(E) 20 1%
Omit 64%

(33) The shaded crescent area is bounded by the semicircle $\overset{\frown}{ADB}$ with diameter $\overline{AB}$ and by the arc $\overset{\frown}{ACB}$ of a circle with center P. If $AB = 12$ and $AP = 12$, what is the area in inches of the shaded crescent area?

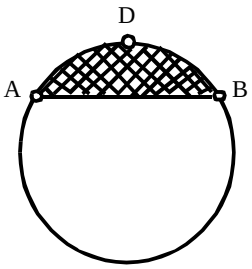


(A) $\boxed{36\sqrt{3} - 6\pi}$ 20%
(B) $18\sqrt{3} - 4\pi$ 6%
(C) 22π 3%
(D) $36\sqrt{3}\pi$ 4%
(E) 40π 0%
Omit 66%

(34) How many sides has a regular polygon each interior angle of which measures 160°?

(A) 12 6%
(B) 18 60%
(C) 20 7%
(D) 24 1%
(E) 36 1%
Omit 25%

(35) Find the perimeter of the shaded area $ADBA$, if the radius of the circle is 12 and $m\overset{\frown}{ADB} = 120°$.



(A) 12π 3%
(B) $4\pi + 12\sqrt{3}$ 6%
(C) 18π 3%
(D) $8\pi + 6\sqrt{3}$ 4%
(E) $\boxed{8\pi + 12\sqrt{3}}$ 25%
Omit 59%

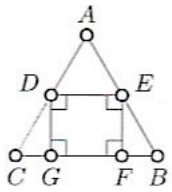
(36) The vertices of a triangle $\angle ABC$ are $A = (-3, 2)$, $B = (6, 5)$, and $C = (9. - 4)$. Find the length of the median from B to side $\overline{AC}$.

(A)	$2\sqrt{5}$	**(B)**	3	**(C)**	$\boxed{3\sqrt{5}}$	**(D)**	4
	3%		3%		34%		5%
(E)	$5\sqrt{5}$	Omit					
	2%	53%					

(37) The sum of the areas of two regular decagons is 39 square inches, and their radii are in the ratio 2 : 3. Find the area of the larger decagon.

(A)	12	**(B)**	17	**(C)**	22	**(D)**	$\boxed{27}$
	3%		4%		6%		21%
(E)	29	Omit					
	1%	65%					

(38) In the figure, triangle $\angle ABC$ is an equilateral triangle. The points D and E are the midpoints of sides $\overline{AC}$ and $\overline{AB}$. respec–tively. What is the ratio of the area of the quadrilateral $\square EFG$ to the area of triangle $\angle ABC$?



(A)	$\frac{1}{3}$	**(B)**	$\boxed{\frac{1}{2}}$	**(C)**	$\frac{5}{8}$	**(D)**	$\frac{3}{4}$
	6%		34%		6%		5%
(E)	1	Omit					
	6%	43%					

(39) Find an equation of the perpendicular bisector of the segment joining $(3, -1)$ and $(-1, 7)$.

(A)	$\boxed{x - 2y = -5}$	24%
(B)	$x + 2y = 7$	3%
(C)	$2x + y = -5$	4%
(D)	$2x + y = -5$	13%
(E)	$2x - y = -1$	5%
Omit		51%

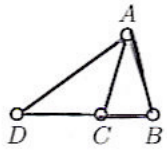
(40) What is the volume of a regular square pyramid with base edge 16 cm and height 6 cm?

(A)	144 cm^3	**(B)**	216 cm^3	**(C)**	336 cm^3
	3%		4%		4%
(D)	$\boxed{512\ \text{cm}^3}$	**(E)**	625 cm^3	Omit	
	46%		1%	42%	

(41) A car travels North for 4 miles, then West for 6 miles, and then South West for 2 miles. How many miles is the car from its starting point?

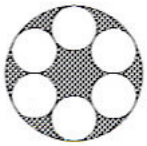
(A)	$\boxed{\sqrt{56+4\sqrt{2}}}$	14%
(B)	$\sqrt{16+2\sqrt{2}}$	2%
(C)	$\sqrt{36+4\sqrt{2}}$	8%
(D)	$\sqrt{6+2\sqrt{2}}$	5%
(E)	$\sqrt{12+4\sqrt{2}}$	5%
Omit		67%

(42) In the figure $AB = AC = CD$, and $AD = BD$. Find the measure of $\angle ADC$ in degrees.



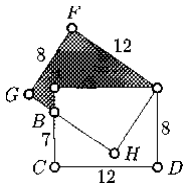
(A)	28°	**(B)**	32°	**(C)**	$\boxed{36°}$	**(D)**	54°
	5%		8%		28%		5%
(E)	72°	Omit					
	2%	52%					

(43) Given six congruent circles drawn internally tangent to a circle of radius 18; each smaller circle is also tangent to each of its adjacent circles. Find the shaded area between the large circle and the six smaller circles.



(A)	36π	**(B)**	96π	**(C)**	$\boxed{108\pi}$
	2%		4%		23%
(D)	144π	**(E)**	216π	Omit	
	4%		3%	63%	

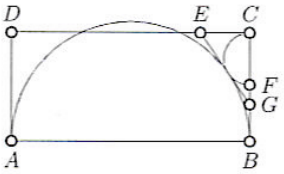
(44) In the given figure, the two rectangles $\square EFGH$ and $\square ACDE$ share a common corner at E and overlap so that BC = 7. What is the area of the shaded region $\square ABGFEA$?



(A) 36 5% **(B)** 54 19% **(C)** 64 5% **(D)** 72 2%

(E) 96 3% Omit 66%

(45) The figure $\square ABCD$ is a rectangle. $AD = 6$, $AB = 18$, arc $\overset{\frown}{AB}$ is a semicircle with diameter $\overline{AB}$, arc $\overset{\frown}{CF}$ is a semicircle with diameter $\overline{CF}$, and $\overline{EG}$ is tangent to both semicircles. What docs EC =?



(A) $\frac{1}{2}$ 2% **(B)** $\frac{2}{3}$ 4% **(C)** $\frac{3}{4}$ 5%

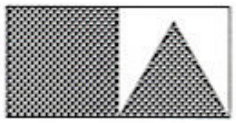
(D) 1 3% **(E)** 2 81% Omit 79%

(46) If one were to cut five circles, each 2 cm in diameter, from a rectangular piece of paper 6 cm long, at least how many cm's wide would the piece of paper have to be?

(A) $\sqrt{3}$ 3% **(B)** $2+\sqrt{3}$ 13% **(C)** 4 26%

(D) $3\sqrt{3}$ 7% **(E)** 2 5% Omit 46%

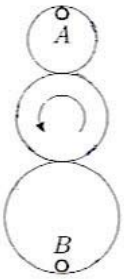
(47) A square of side length S and an equilateral triangle of side length S are placed in side a rectangle of length $2S$ and width S as shown. What fraction of the area of the rectangle remains uncovered?



(A) $\frac{4+\sqrt{3}}{8}$ 7% **(B)** $\frac{1+2\sqrt{3}}{2}$ 6% **(C)** $\boxed{\frac{4-\sqrt{3}}{8}}$ 14%

(D) $\frac{1-2\sqrt{3}}{2}$ 3% **(E)** $\frac{2+10\sqrt{3}}{4}$ 2% Omit 67%

(48) Three friction gears are shown below. They turn without any slippage and the centers of the gears are in line with one another. The diameters of the friction gears are. $6, 8,$ and 10, respectively. As the center friction gear rotates counter clockwise, it causes the top and bottom friction gear to rotate clockwise. The top and bottom friction gears each have a timing mark stamped on the friction gear, which of course rotate as the gear rotates. Timing mark A is initially at top dead center (at the very top of the friction gear), while timing mark B is at bottom dead center (at the very bottom of the friction gear). What is the minimum number of degrees that the middle friction gear must rotate in order for timing mark A to be at bottom dead center while timing mark B will be at top dead center?



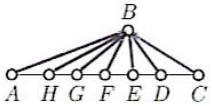
(A) 180° 14% **(B)** 900° 4% **(C)** 405° 4%

(D) 540° 8% **(E)** $\boxed{675°}$ 9% Omit 60%

(49) Given a pair of different lines which intersect one another, and a circle which does not intersect either line; how many different circles can be drawn which are tangent to both lines, and also tangent to the given circle?

(A)	None	2%
(B)	Always exactly one	17%
(C)	Always exactly two	14%
(D)	Either none or exactly one depending on on the position of the circle and the lines	12%
(E)	Either exactly one or exactly two depending on the position of the circle and the lines	9%
Omit		46%

(50) In the given figure, $AB = 6$, $BC = 4$, $CD = 1$, $CE = 1.8$, $CF = 3$, $CG = 3.6$, $CH = 4.5$, and $CA = 9$. Which segment bisects angle $\angle ABC$?



(A)	$\overline{BD}$	**(B)**	$\overline{BE}$	**(C)**	$\overline{BF}$	**(D)**	$\overline{BG}$
	0%		7%		30%		10%

(E)	$\overline{BH}$	Omit
	21%	31%